20U313S (Pages: 2) Name: …….……..…….

Reg. No: …….……..…….

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

CC15U MAT3 B03/ CC18U MAT3 B03 - CALCULUS AND ANALYTIC GEOMETRY

(Mathematics - Core Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

Part-A

Answer ***all*** questions. Each question carries 1 mark.

1. Evaluate $e^{3\,ln2}$
2. Find $\lim_{x\to 0}\frac{e^x-1}{x}$
3. The hyperbolic sine of x is defined as ……
4. Define monotone sequence.
5. Check the convergence of $\sum_{n=1}^{\infty}\ n^2$
6. State ratio test.
7. The Taylor series expansion of $f(x)$ about a point $x = a$ is …….
8. Find a formula for the n^{th} term of the sequence $1, -1, 1, -1, \ldots\ldots\ldots\ldots\ldots$
9. Define absolute convergence test.
10. Find the vertices of the ellipse $2x^2 + 3y^2 = 6$.
11. Write the parametric equation of the circle $x^2 + y^2 = 1$.
12. Graph the set of points whose polar coordinates satisfy the condition $0 \le r \le 1$.

(12 × 1 = 12 Marks)

Part B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Evaluate $\int_0^{\pi/2} e^{\sin x}\ cosx\ dx$
14. Find $\frac{dy}{dx}$ if $y = x^x,\ \ x > 0$.
15. Evaluate $\min_{x\to 0} x^x$
16. Evaluate $\int_0^{ln2} 4e^x \sinh x\ dx$.
17. Evaluate $\lim_{n\to\infty} \ln\,(1+\frac{1}{n})^n$.
18. Test the convergence of $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$
19. Test the convergence of $\sum_{n=1}^{\infty}\frac{n^n}{n!}$.

20. Use the discriminant to decide whether $x^2 - 3xy + y^2 - x = 0$ is parabola.
21. Find the equation of the hyperbola with foci $(0, \pm\sqrt{2})$ and asymptotes $y = \pm x$.
22. Replace the cartesian equation $y^2 = 4x$ by equivalent polar equation.
23. Find the slope of the curve $x = 4\ sint$ and $y = 2\cos t$.
24. Polar equation of a conic is $r = \frac{12}{3+3\ sin\theta}$. Identify the conic.

(9 × 2 = 18 Marks)

Part C

Answer any ***six*** questions. Each question carries 5 marks.

25. Given $\sinh x = -\frac{3}{4}$. Find the other five hyperbolic functions.
26. If x is a real, show that $cosh^{-1}x = \log\ (x + \sqrt{x^2 - 1}\ )$
27. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \cdots$
28. Test the convergence of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \cdots (0 < x < 1)$
29. By a suitable rotation of the rectangular axes about the origin, remove the xy term in $5x^2 - 6xy + 5y^2 = 8$.
30. Find the tangent to the curve $x = 4\sin t$ and $y = 2\cos t$ at $t = \frac{\pi}{4}$. Also find the value of $\frac{d^2y}{dx^2}$ at this point.
31. Find the area of the surface generated by revolving the curve whose parametrization is $x = t + \sqrt{2}, \quad y = \frac{t^2}{2} + \sqrt{2}\ t, \quad -\sqrt{2} \le t \le \sqrt{2}$ about the $y - axis$
32. Graph the curve $r = 1 - \cos\theta$.
33. Find the area of the curve $r^2 = a^2\cos 2\theta$

(6 × 5 = 30 Marks)

Part D

Answer any ***two*** questions. Each question carries 10 marks.

34. Show that the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$converges to $tan^{-1}x$ for all $-1 < x < 1$.
35. Find the length of the astroid $x = cos^3t,\ y = sin^3t, \quad 0 \le t \le 2\pi$. Also find the centroid of the first quadrant arc of the above astroid.
36. Define eccentricity of a conic section. Clarify the conic section by eccentricity. How are an ellipses shape and eccentricity related?

(2 × 10 = 20 Marks)

****