20P301 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C11 - MULTIVARIABLE CALCULUS AND GEOMETRY

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Check whether $f: \mathbb{R}^3 \to \mathbb{R}^4$ defined by $f(x, y, z) = (x, y, 1 + x, 1 + y^2)$ is a linear transformation or not. Justify your answer.
2. Define contraction map with an example
3. Let Ω be the set of all linear operators on $\mathbb{R}^n$. Let $A \in \Omega$ and $B \in L(\mathbb{R}^n)$ with $\|B - A\|. \|A^{-1}\| < 1$. Prove that B ∈ Ω.
4. Define arc length of a curve γ and find the length of the logarithmic spiral $\gamma(t) = (e^{kt} \cos t, e^{kt} \sin t)$ starting at $(1, 0)$
5. What you mean by the derivative of a smooth map between two surfaces.
6. Prove that the surface patch $\sigma(u, v) = (\cos u, \sin u, v)$ for a circular cylinder is regular.
7. Prove that $\kappa^2 = \kappa_n{}^2 + \kappa_g{}^2$
8. Define parabolic point of a surface.

(8 × 1 = 8 Weightage)

PART B

Answer any ***six*** questions. Each question carries 2 weightage.

UNIT I

9. State and prove Chain Rule for functions of several variables.
10. Suppose that $f: E \to \mathbb{R}^m$ where $E \subset \mathbb{R}^n$ is open. Prove that $f \in C^1(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq n$.
11. Show that if $[A]$ and $[B]$ are $n \times n$ matrices then $det[A][B]$= $det[A]det[B]$

UNIT II

12. Define torsion of a parametrized curve and derive a formula for the same for any regular curve with nowhere vanishing curvature.
13. Prove that the transition maps of a smooth surface are smooth.

14. Let $k:(\alpha,\beta) \to \mathbb{R}$ be any smooth function. Prove that there exists a unit speed curve whose curvature is k. Also prove that if there is any other curve with same signed curvature, then both of these curves are direct isometries of each other.

UNIT III

15. Let S and $\tilde{S}$ be surfaces and let $f : S \to \tilde{S}$ be a smooth map. Then, prove that f is a local diffeomorphism if and only if, for all $p \in S$, the linear map $Dpf : T_pS \to T_{f(p)}\tilde{S}$ is invertible.
16. State and prove Meusnier's Theorem.
17. State and prove Euler's Theorem.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove Inverse Function Theorem
19. (a) Define a surface in $\mathbb{R}^3$and prove in detail that (i)a plane (ii) the unit sphere S^2 are surfaces.

    (b) Give an example of a subset in $\mathbb{R}^3$ which is not a surface. Justify your answer.
20. (a) Find the First and Second fundamental forms of the unit sphere S^2.

    (b) Find the First fundamental form of the surface of revolution whose surface patch can be given by $\sigma\,(u,v) = (f(u)\,cos\,v, f(u)\,sin\,v, g(u))$ where $(f(u), 0, g(u))$ is the profile curve
21. Let S be a connected surface of which every point is an umbilic. Then prove that S is an open subset of a plane or a sphere.

(2 × 5 = 10 Weightage)

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