20P302 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C12 - COMPLEX ANALYSIS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define the radius of convergence of a given power series.
2. When is S, a Mobius transformation, called a translation?
3. State the conditions for two rectifiable paths to be equivalent.
4. Find $\int_{\gamma} \frac{\sin z}{z^3} dz$, where $\gamma(t) = e^{it}$, $0 \leq t \leq 2\pi$.
5. State and prove open mapping theorem.
6. When will you say two curves are fixed end point homotopic?
7. Define an essential singularity. Give an example.
8. State Schwarz's Lemma.

(8 × 1 = 8 Weightage)

Part B

Answer any ***six*** questions. Each question carries 2 weightage.

UNIT 1

9. Show that $f(z) = |z|^2 = x^2 + y^2$ has a derivative only at the origin.
10. State and Prove Symmetry Principle.
11. Prove that a Möbius transformation takes circles onto circles.

UNIT 2

12. Derive the Cauchy's estimate.
13. State and prove Morera's theorem.
14. Evaluate the integral $\int_{\gamma} \frac{e^z - e^{-z}}{z^n} dz$, where n is a positive integer and

$$\gamma(\theta) = e^{i\theta}, o \leq \theta \leq 2\pi$$

UNIT 3

15. State and prove Argument Principle.
16. State and prove Casorati-Weierstrass theorem.
17. State and prove Rouche's theorem. Derive fundamental theorem of algebra using Rouche's theorem.

$(6 \times 2 = 12$ Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let $u \; and \; v$ be real valued functions defined on a region G and suppose that $u \; and \; v$ have continuous partial derivatives. Then $f: G \to \mathbb{C}$ defined by $f(z) = u(z) + i\, v(z)$ is analytic iff $u \; and \; v$ satisfy the Cauchy Reimann equations.
19. Derive Cauchy's integral formula. Make a note on the significance of the first and second versions of the same.
20. State and prove Goursat's Theorem.
21. Evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$ by using Residue Theorem.

$(2 \times 5 = 10$ Weightage)

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