20P304 (Pages: 2) **Name: ....**

Reg No: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C14 - PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2019 Admission onwards)

Time: 3:00 Hours **Maximum: 30 Weightage**

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that $u_x + u_y = 1, u(x, x) = x$ has infinitely many solutions.
2. Define eikonal equation.
3. Classify different types of second order partial differential equations and give examples for each
4. Find the characteristics of Tricomi equation:$u_{xx} + xu_{yy} = 0, x < 0$.
5. State and prove necessary condition for the existence of a solution to the Neumann problem.
6. State and prove strong maximum principle
7. Distinguish between Fredholm Equation and Volterra equation
8. Define separable kernal with an example

($8 \times 1 = 8$ Weightage)

PART B

Answer any ***six*** questions. Each question carries 2 weightage.

Unit I

9. Solve the the Cauchy problem $u_x + u_y = u^2$, $u(x, 0) = 1$ using method of characteristic
10. Prove that the type of a linear second-order PDE in two variables is invariant under a change of coordinates.
11. Let $u(x, t)$ be the solution of the Cauchy problem. $u_{tt} - 9u_{xx} = 0, \quad -\infty < x < \infty, t > 0,$
$u(x, 0) = f(x)$
$u_t(x, 0) = g(x)$
where $f(x) = 1$ if $|x| \leq 2$ and $f(x) = 0$ if $|x| > 2$, and $g(x) = 1$ if $|x| \leq 2$ and $g(x) = 0$ if $|x| > 2$.
(a) Find $u(0, 16)$.
(b) Discuss the large time behavior of the solution.

Unit II

12. Consider the equation $xu_{xx} - yu_{yy} + 12(ux - uy) = 0$.Find the domain where the equation is elliptic and find the corresponding canonical transformation in that domain.
13. Let u be a function in $C^2(\mathbf{D})$ satisfying the mean value property at every point in $\mathbf{D}$. Then u is harmonic in $\mathbf{D}$

14. Solve the Laplace equation in the rectangle $0 < x < b, 0 < y < d$, subject to the Dirichlet boundary conditions $u(0, y) = f(y), u(b, y) = g(y), u(x, 0) = 0, u(x, d) = 0$.

Unit III

15. Prove that the characteristic numbers of Fredholm equation with real symmetric kernal are all real

16. Transform the problem $\dfrac{d^2y}{dx^2} + xy = 1$ with $y(0) = 0, y(1) = 0$ to integral equation $y(x) = \displaystyle\int_0^1 G(x, \xi)\xi y(\xi)d\xi - \frac{1}{2}x(1-x)$ where $G(x, \xi) = \begin{cases} x(1-\xi) & ; x < \xi \\ \xi(1-x) & ; x > \xi \end{cases}$

17. Determine the resolvant kernal of $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)y(\xi)d\xi$ Hence find the solution.

($6 \times 2 = 12$ Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Define transversality condition. Give one example in which it fails.
(b) Solve the equation $-yu_x + xu_y = u$ subject to the initial condition $u(x, 0) = \psi(x)$

19. Find general solution and D'Alembert's solution of Cauchy problem for the one-dimensional non-homogeneous wave equation.

20. Find Fourier series solution using separation of variable method for the problem
$u_t - u_{xx} = 0, 0 < x < \pi, t > 0;$
$u_x(0, t) = u_(\pi, t) = 0, t \geq 0;$
$u(x, 0) = f(x) = \begin{cases} x & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \leq x \leq \pi \end{cases} , 0 \leq x \leq L$

21. If $y'' = F(x)$ and y satisfies the initial conditions $y(0) = y_0$, $y'(0) = y'_0$
(a) Show that $y(x) = \int_0^x (x - \xi)F(\xi)d\xi + xy'_0 + y_0$
(b) Verify that the expression satisfies the prescribed differential equation and the initial condition.

($2 \times 5 = 10$ Weightage)

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