THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P ST3 C12 - STOCHASTIC PROCESSES

(Statistics)

(2019 Admission onwards)

Time: Three Hours   Maximum: 30 Weightage

PART A

Answer any *four* questions. Each question carries 2 weightage.

1. Prove that Markov chain is completely determined by the one-step TPM and the initial distribution.

2. Show that state i is recurrent if $\sum_{n=1}^{\infty} p_{ii}^{(n)} = \infty$ and is transient if $\sum_{n=1}^{\infty} p_{ii}^{(n)} < \infty$.

3. Distinguish between open and closed sets.

4. Bring out the relation between Poisson process and Binomial distribution.

5. Derive the Chapman-Kolmogorov equation.

6. Explain Stationary distribution with the help of an example.

7. Derive Poisson Process.

(4 × 2 = 8 Weightage)

PART B

Answer any *four* questions. Each question carries 3 weightage.

8. (a) Derive Yule-Furry Process.

    (b) Show that inter arrival times are exponentially distributed.

9. (a) Stochastic process having independent increment is a Markov process. Is the converse true, justify?

    (b) Show that the number of renewals by time $t \geq n$ if and only if the nth renewal occurs on or before time t.

10. (a) Explain Brownian motion process.

    (b) Derive the distribution of first hitting time of Brownian motion process.

11. (a) Explain Ergodic theorem

    (b) Derive Wald's equation

12. (a) Define Birth and death process.

    (b) Derive the steady state probabilities of M/M/1 model.

13. (a) Show that the renewal function $m(t) = \sum_{n=1}^{\infty} F_n(t), \forall t,$

      where $F_n(t) = P(S_n \leq t), n \geq 1, \forall\ t.$

    (b) Write down the steady state equations of Erlang's Loss system.

14. Check whether the following Markov chain with four states 0, 1, 2, and 3 having TPM

    given below is ergodic $\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$

(4 × 3 = 12 Weightage)

PART C

Answer any *two* questions. Each question carries 5 weightage.

15. (a) Derive Pollock-Kinchins formulae.

    (b) What do you mean by queue? Briefly explain Kendall's Notation

16. (a) State and prove central limit theorem on renewal process.

    (b) Define Stochastic processes and its various states with the help of examples.

17. (a) Establish the relation between probability generating functions of off spring random

      variable and n^{th} generation size in Galton –Watson branching Process.

    (b) Explain the regenerative stochastic process and Semi Markov process

18. (a) Explain the transient behaviour of M/M/S model.

    (b) Derive the limiting probabilities of a Birth-Death process.

(2 × 5 = 10 Weightage)

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