18U604 (Pages: 2) Reg. No...............................
Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021
(CUCBCSS-UG)
(Regular/Supplementary/Improvement)
CC15U MAT6 B12/CC18U MAT6 B12 -
NUMBER THEORY AND LINEAR ALGEBRA
(Mathematics - Core Course)
(2015 Admission onwards)

Time: Three Hours
Total : 120 Marks

Part A
Answer *all* questions. Each question carries 1 mark.

1. Write two numbers which are prime to each other.

2. Prove or disprove: If $a|b$ and $a|c$ then $a^2|bc$.

3. Write a linear Diophantine equation.

4. Express 360 in the canonical form.

5. Translate $(1001111)_2$ to decimal system.

6. State Wilson's theorem

7. Find $\tau(18)$.

8. Give an example for a multiplicative function.

9. Define Euler's phi-function.

10. Give an example for a 2 dimensional vector space.

11. Find a basis for $\mathbb{R}_2[x]$ over $\mathbb{R}$

12. Give an example for a linear map from $\mathbb{R}$ to $\mathbb{R}^2$.

(12 × 1 = 12 Marks)

Part B
Answer any *ten* questions. Each question carries 4 marks.

13. Show that the square of any odd integer is of the form $8k + 1$.

14. Find the $gcd(26, 382)$ and express it as the linear combination of 26 and 382.

15. Find all integer solutions of $14x + 16y = 15$.

16. True or false: The number of primes is finite. Justify your answer.

17. Find all prime numbers that divide 50!

18. Find the remainder obtained when $1! + 2! + 3! + \cdots + 99! + 100!$ is devided by 12.

19. Show that 8th power of any number is of the form $17m$ or $17m \pm 1$.

20. Find the number and sum of divisors of 4116.

21. Find the number of zeros with which the decimal representation of 50! terminates.

22. Find all subspaces of $\mathbb{R}^3$ over $\mathbb{R}$.

23. Check whether $(2, 6, -4), (3, 9, -6)$ are linearly independent in $\mathbb{R}^3$.

24. Define dimension of a vector space.Give example of a vector space of infinite dimension.

25. Show that $f^{\rightarrow}$ is inclusion preserving, if $f : U \longrightarrow V$ is a linear map.

26. Find $Ker\ f$ where f is the ith projection map from $\mathbb{R}^n$ to $\mathbb{R}$.

(10 × 4 = 40 Marks)

Part C
Answer any **six** questions. Each question carries 7 marks.

27. Determine all solutions of the Diophantine equation $54x + 21y = 906$.

28. Compute the remainder when 3^{247} is divided by 25.

29. Solve the linear congruence $12x \equiv 48(\mod 18)$.

30. Find the remainder when 18! is divided by 23.

31. Let V be a vector space and U, W are subspaces of V. Prove that $U \cap W$ is a subspace of V. Check whether $U \cup W$ is a vector space.

32. Define linear independent set. Show that no linear independent subset of a vector space V can contain O_V.

33. Show that a linear map $f : U \longrightarrow V$ is injective if and only if $Ker\ f = \{0_U\}$.

34. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ be $T(x_1, x_2, x_3) = (x_1, x_1 + x_2, x_1 + x_2 + x_3, x_3)$. Verify dimension theorem for T.

(6 × 7 = 42 Marks)

Part D
Answer any **two** questions. Each question carries 13 marks.

35. Solve the linear congruence $17x \equiv 9 \pmod{276}$.

36. State and prove Fermat's little theorem. Is the converse of the Fermat's little theorem true? Justify your answer.

37. State and Prove Dimension theorem.

(2 × 13 = 26 Marks)

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