19U407S (Pages: 2) Name ……………………….

Reg. No. …………….……..

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCBCSS - UG)

CC15U MAT4 C04 - MATHEMATICS - IV

(Mathematics - Complementary)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark.

1. Write the general form Euler Cauchy equation.
2. Define a periodic function give an example
3. Write the characteristic equation of $y'' + ay' + by = 0$.
4. Prove that $L(1) = \frac{1}{s}, s > 0$.
5. What is the Laplace transform of $\sinh at$?
6. If $f(t) = 0$, find $\mathcal{L}(0)$.
7. State Linearity property of Laplace transform.
8. Define unit step function
9. Is the function $x \cos nx$ odd, even neither odd nor even?
10. Write two dimensional wave equation.
11. Write the iteration formula for the Picard's method.
12. What is the error term in trapezoidal rule?

(12 × 1 = 12 Marks)

PART B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Find the Wronskian of e^{ax} and e^{bx}
14. Find the general solution of $y'' + 2y' + 5y = 0$
15. Apply the operator (D-2)(D+1) on $y = xe^{2x}$.
16. Find $a_0 and$ a_n of the Fourier series $f(x) = \left\{\begin{matrix} k & -\pi < x < 0 \\ -k & 0 < x < \pi \end{matrix}\right\}$
17. Find the Laplace transform of $e^{-3t} \sin^2 t$.
18. Find the Laplace transform of $(t+1)^2 e^t$.
19. Prove that $\mathcal{L}(cosat) = \frac{s}{s^2+a^2}$.

20. Show that $f * g = g * f$ where $f * g$ denote the convolution of two functions f and g.

21. Compute y_3 by Euler method with h=0.2 for IVP $y'=x+y$,$y(0)=0$

22. Solve the system of partial differential equation $u_{xx} = 0,\ u_{yy} = 0$.

23. Find $y_2(x)$ for initial value problem $y'=1+y^2$,$y(0)=0$ by Picard's method.

24. Use Trapezoidal rule with $n = 4$ to estimate $\int_0^2 \frac{1}{1+x} dx$. Also find the upper bound for error in the above approximation.

(9 × 2 = 18 Marks)

PART C

Answer any ***six*** questions. Each question carries 5 marks.

25. Find the general solution of the differential equation $y'' - 2y' = 12x - 10$

26. Solve the following initial value problem

$4x^2y'' + 24xy' + 25y = 0,\ \ y(1) = 2,\ \ y'(1) = -6$

27. Solve $(D^2 - 2D + 1)y = 10\ e^x sinx$

28. Find the inverse Laplace transform of $\frac{5s^2-15s-11}{(s+1)(s-2)^3}$.

29. Find solutions $u(x, y)$ of the function $u_x = 2\ u_y + u$ by separating variables.

30. Show that $\mathcal{L}^{-1}\left[\ln\left(1 + \frac{\omega^2}{s^2}\right)\right] = \frac{2}{t}(1 - cos\omega t)$.

31. Solve the integral equation $y(t) = 1 + \int_0^t y(t)dt$

32. Find the half range sine series of the function $f(x) = \begin{cases} x, & if\ \ 0 < x < \frac{\pi}{2} \\ \pi - x, & if\ \frac{\pi}{2} \le x < \pi \end{cases}$

33. Divide range into 10 equal parts to find $\int_0^\pi sinx\ dx$ using Simpsons rule.

(6 × 5 = 30 Marks)

PART D

Answer any ***two*** questions. Each question carries 10 marks.

34. Using the Laplace transformation solve the initial value problem

$$y'' + 2y' - 3y\ = sint,\ \ y(0) =\ 0, y'(0) =\ 0.$$

35. Find the inverse Laplace transform of (a) $\frac{1}{(s^2+\omega^2)^2}$ (b) $ln\frac{s^2+1}{(s-1)^2}$

36. Find the Fourier series of the function $f(x) = \pi \sin \pi x$, $0 < x < 1$ with period $2p = 1$.

37. Using Runge-Kutta method Solve IVP $y' = x + y$, $y(0) = 0$, $h = 0.2\ for\ x = 1$

(2 × 10 = 20 Marks)

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