20P259 (Pages: 2) Name…………………………

Reg. No………………………

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST2 C07 - ESTIMATION THEORY

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define completeness. Give an example of a family of distributions which is not complete.
2. Let X be a random variable with p.d.f.

$$f(x) = \begin{cases} \frac{1}{2}; \theta < x < \theta + 2 \\ 0; otherwise \end{cases}$$

where θ is any real number? Examine whether there is any statistic sufficient for θ.

3. Define Fisher information. Let $x_1, x_2, \ldots\ldots x_n$ be i.i.d. radom variables which follows Poisson distribution with parameter λ. Find Fisher information.
4. Define CAN estimator. Show that $\bar{X}$ is CAN for θ in Poisson (θ)
5. Define ancillary statistics. Discuss its role in estimation.
6. Describe the pivotal quantity method of constructing shortest confidence interval.
7. Find Cramer -Rao lower bound for any unbiased estimator of θ based on *n* i.i.d observations from the population with p. d. f.

$$f(x) = \frac{1}{\pi(1+(x-\theta)^2)}, -\infty < x < \infty, -\infty < \theta < \infty$$

(4 × 2 = 8 Weightage)

Part B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Let $X_1, X_2, \ldots, X_n$ be i. i. d. observations from Poisson P(λ). Find a consistent estimator of $e^{-\lambda}$.
9. Let $X_1, X_2, \ldots, X_n$ be i. i. d. observations from $U(\theta,\ \theta+1)$, Show that $\left(X_{(1)}, X_{(n)}\right)$ is minimal sufficient for θ, but not complete.
10. Prove or disprove the statement Maximum Likelihood Estimators need not be unique.
11. State and prove Lehmann Scheffe theorem.

12. Define UMVUE. Are they unique? Justify your answer.
13. State and prove sufficient condition for consistency of an estimator.
14. Define one parameter exponential family of distributions. Give an example of a distribution which is not a member of this family.

($4 \times 3 = 12$ Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

15. State Basu's theorem. Applying Basu's theorem show that sample mean and sample variance are independently distributed if the sample is drawn from a normal population with mean μ and variance σ^2.
16. Based on a sample drawn from $N(\mu, \sigma^2)$, with σ^2 unknown, construct a $100(1-\alpha)\%$ confidence interval for μ.
17. State and prove asymptotic properties of maximum likelihood estimators.
18. (a) State and prove Rao-Blackwell theorem.

    (b) Let $x_1, x_2, \dots \dots x_n$ be a random sample sequence from N(0, 1). Find UMVUE of θ and θ^2

($2 \times 5 = 10$ Weightage)

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