20P202 (Pages: 2) Name: …..…………………….

Reg. No: ….…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C07 – REAL ANALYSIS II

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum:30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Lebesgue outer measure of a set. Give an example
2. Define $\sigma - Algebra$ with example.
3. Let G be the set of irrational numbers in the interval $[1,3]$. Show that $m^*(G) = 2$.
4. Show that real valued continuous functions on measurable domains are measurable.
5. Define Lebesgue integral.
6. Establish the existence of a function which is Lebesgue integrable but not Riemann integrable.
7. Define convergence in measure of a sequence $\{f_n\}$ of measurable functions. Give example.
8. Establish the existence of a function which is not of bounded variation on $[a, b]$.

(1 × 8 = 8 Weightage)

PART B

Answer any ***six*** questions. Each question carries 2 weightage.

UNIT I

9. State and prove Heine-Borel theorem.
10. Prove that the union of a countable collection of measurable sets is measurable.
11. Let f be a measurable function on E which is finite a.e on E. Prove that f^2 is measurable on E.

UNIT II

12. State and prove Chebychev's inequality.
13. State and prove Fatou's lemma.
14. Let f be a bounded function on a set of finite measure E. Show that f is Lebesgue integrable over E if and only if it is measurable.

UNIT III

15. Show that there is a strictly increasing function on $[0,1]$ that is continuous only at the irrational numbers in $[0,1]$.
16. Find the upper and lower derivatives of $f(x) = |x|$ at $x = 0$.
17. Prove Cauchy-Schwarz Inequality.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) State and Prove the Borel- Cantelli lemma.

    b) Establish the existence if Vitali sets.
19. Give the statement and prove the simple approximation theorem.
20. a) State and prove the bounded convergence theorem.

    b) State and prove the Lebesgue dominated convergence theorem.
21. Establish the inequalities of Young, Holder and Minkowski.

(2 × 5 = 10 Weightage)

****