20P204 (Pages: 2) Name……………..……..….

Reg. No………...………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C08 - TOPOLOGY

(Mathematics)

(2019 Admissions onwards)

Time: Three Hours Maximum:30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage

1. Write an example for a subspace of a topological space.
2. Prove that in a metric space union of arbitrary collection of open sets is open.
3. Define usual topology on the set of real numbers.
4. Prove that every continuous image of a compact space is compact
5. Prove that closure of a connected set is connected.
6. Distinguish between absolute property and relative property of a subset of a topological space.
7. Define pointwise convergence and uniform convergence of sequence of functions in a topological space.
8. State Urysohn's lemma.

(8 × 1 = 8 Weightage)

Part B

Answer any ***six*** questions. Each question carries 2 weightage.

UNIT 1

9. Prove that if a space is second countable then every open cover of it has a countable subcover.
10. Prove that metrisability is a hereditary property
11. Define co-countable topology. Prove that in the co- countable topology the only convergent sequences are those which are eventually constant.

UNIT 2

12. Every closed surjective map is a quotient map.
13. Prove that topological product of any finite number of connected space is connected.
14. Prove that quotient space of a locally connected space is locally connected.

UNIT 3

15. Prove that in a Hausdroff space, limit of a sequence is unique.
16. Prove that continuous bijection from a compact space onto a Hausdroff space is a homeomorphism
17. Prove that a metric space is a T_3 space.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Let X be a set, τ a topology on X and **S** a family of subsets of X. Prove that **S** is a sub-base for τ if and only if **S** generates τ

    b) Let $Z \subset Y \subset X$ and τ be a topology on X. Then with usual notations prove that $(\tau/Y)/Z = \tau/Z$.
19. a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema

    b) State and prove Lebesque covering lemma.
20. a) Prove that every regular, Lindeloff space is normal.

    b) Suppose y is an accumulation point of a subset A of a T_1 space. Prove that every neighbourhood of y contains infinitely many points of A
21. State and prove Tietze Characterisation of Normality.

(2 × 5 = 10 Weightage)

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