20P205 (Pages: 2) Name: ……….……………

Reg. No. ………………….

SECOND SEMESTER M. Sc. DEGREE EXAMINATION, APRIL 2021

(CUCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C09 – ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Marks: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find a power series solution of the equation $y' = y$.
2. Verify that $p_n(-1) = (-1)^n$, where $p_n(x)$ is the n^{th}degree Legendre polynomial.
3. If $y(x)$ be a nontrivial solution of equation $y'' + q(x)y = 0$ on a closed interval $[a, b]$. Then $y(x)$ has atmost a finite number of zeros in this interval.
4. Describe the phase portrait of the system: $\frac{dx}{dt} = 1, \frac{dy}{dt} = 2$.
5. Show that $\begin{Bmatrix} x = e^{4t} \\ y = e^{4t} \end{Bmatrix}$ and $\begin{Bmatrix} x = e^{-2t} \\ y = -e^{-2t} \end{Bmatrix}$ are solutions of the homogeneous system $\begin{Bmatrix} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{Bmatrix}$
6. Using Picard's method, find the solution of $y' = y$ with the initial condition $y(0) = 1$.
7. Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p\, J_{p-1}(x)$.
8. Determine whether the function $2x^2 - 3xy + 3y^2$ is positive definite, negative definite or neither.

(8 x 1 = 8 Weightage)

Part B

Answer any ***six*** questions. Each question carries 2 weightage.

Unit I

9. Derive the orthogonality property of the Legendre Polynomial.
10. Find the general solution of $y'' + xy' + y = 0$.
11. Find two independent Frobenius series solutions of the equation $2xy'' + (3 - x)y' - y = 0$.

Unit II

12. If $W(t)$ is the Wronskian of the two solutions of the homogeneous linear system $\left\{ \begin{array}{l} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{array} \right\}$, then $W(t)$ is either identically zero or nowhere zero on $[a, b]$.
13. State Bessel Expansion theorem and compute the Bessel series of the function $f(x) = 1$ for the interval $0 \leq x \leq 1$ in terms of the function $J_0(\lambda_n x)$, where λ_n's are the positive zeros of $J_0(x)$.
14. Find the critical points, differential equation of the path, general solution and sketch the path of the system $\frac{dx}{dt} = x; \frac{dy}{dt} = -x + 2y$.

Unit III

15. State and Prove Sturm comparison theorem.
16. Formulate the problem of finding the curve of quickest descent.
17. Find the exact solution of the initial value problem $y' = 2x(1 + y),\ y(0) = 0$. Starting with $y_0(x) = 0$, calculate $y_1(x), y_2(x), y_3(x), y_4(x)$ using Picard's method and compare these results with the exact solution.

(6 x 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Derive the Rodrigue's formula for Legendre polynomials $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.
19. If there exists a Liapunov fuction $E(x, y)$ for the autonomous system, then the critical point (0,0) is stable. Furthermore, if this function has the additional property that the function $\frac{\partial E}{\partial x} F + \frac{\partial E}{\partial y} G$ is negative definite, then the critical point (0,0) is asymptotically stable.
20. State and Prove Picard's theorem.
21. Determine the general solution of Gauss Hypergeometric equation near $x = 0$.

(2 x 5 = 10 Weightage)

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