20P206 (Pages: 2) Name: ………..……………

Reg. No: …………..……….

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

(CUCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C10 - OPERATIONS RESEARCH

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A (Short Answer question)

Answer ***all*** questions. Each question has 1 weightage

1. Prove that the sum of two convex functions is again a convex function.
2. What is the canonical form of equations in linear programming problem?
3. Write the dual of the linear programming problem: Maximise $x_1 + 6x_2 + 4x_3 + 6x_4$ subject to

   $2x_1 + 3x_2 + 17x_3 + 80x_4 \leq 48, \quad 8x_1 + 4x_2 + 4x_3 + 4x_4 = 2$,

   $x_1, x_2 \geq 0;\ x_3$ *and* x_4 *are unrestricted in sign.*
4. What are simplex multipliers in a linear programming problem?
5. What is meant by degeneracy in transportation problem?
6. Prove that the Maximum flow in a graph is equal to the minimum of the capacities of possible cuts in it
7. Describe minimum path problem in network analysis.
8. Describe a two – person zero - sum game.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. State and prove necessary and sufficient conditions for a differentiable function $f(x)$ defined in a convex domain to be a convex function.
10. Prove that a basic feasible solution of a linear programming problem is a vertex of the convex set of feasible solutions.
11. Solve graphically the linear programming problem: Minimize $z = x_1 + 3x_2$ subject to $x_1 + x_2 \geq 3,\ -x_1 + x_2 \leq 2,\ x_1 - 2x_2 \leq 2,\ x_1, x_2 \geq 0$

UNIT II

12. If the primal problem is feasible, then prove that it has an unbounded optimum iff the dual has no feasible solution and vice versa.
13. Prove that the value of the objective function, for any feasible solution of the primal is not less than the value of the objective function for any feasible solution of the dual.
14. Prove that the transportation array has a triangular basis.

UNIT III

15. State and prove fundamental theorem of rectangular games.
16. Describe the generalised problem of maximum flow.
17. Describe cutting plane method to solve an integer linear programming problem.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question has 5 weightage.

15. a) Prove that the dual of the dual of a linear programming problem is primal.

 b) Maximize $f(x) = 5x_1 + 3x_2 + x_3$ subject to constrains $2x_1 + x_2 + x_3 = 3$; $-x_1 + 2x_3 = 4$; $x_1, x_2, x_3 \geq 0$.

16. a) Solve the transportation problem for minimum cost with cost coefficients demands and supplies as in the following table.

	D_1	D_2	D_3	D_4	
O_1	3	2	5	4	25
O_2	4	1	7	6	35
O_3	7	8	3	5	30
	10	18	20	42	

 b) What is Caterer Problem in Operation Research.

17. Solve the integer linear programming problem: Maximize $f(x) = 3x_1 + 4x_2$.
 Subject to $2x_1 + 4x_2 \leq 13$, $-2x_1 + x_2 \leq 2$, $2x_1 + 2x_2 \geq 1$, $6x_1 - 4x_2 \leq 15$, $x_1, x_2 \geq 0$ and x_1 *and* x_2 and integers.

18. (a) Solve graphically the game whose pay-off matrix is $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

 (b) State and prove the min max theorem of game theory.

(2 × 5 = 10 Weightage)

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