FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021
(CUCBCSS-UG)
CC15U MAT1 C01 / CC18U MAT1 C01- MATHEMATICS
(Mathematics Complementary course)
(2016 to 2018 Admission - Supplementary/Improvement)

Time: 3.00 Hours   Maximum: 80 Marks

Part A
Answer **all** questions. Each question carries 1 mark.

1. $\lim_{x \to 0} \dfrac{sinx}{x} = ...?$

2. Does the function $f(x) = |x - 2|$ continuous at $x = 2$?

3. What is the slope of the tangent line to the parabola $y = x - 3x^2$ at the point $(1, -2)$?

4. If $f(x) = \dfrac{1}{x - 2}$, find $f'(2)$.

5. At any time t, the position of a moving particle along s-axis is given by $s = t^2 - 4t$. What is it's velocity when $t = 3$?.

6. What is the absolute minimum value of the function $f(x) = x^2 - 5$ on the interval $(-\infty, \infty)$?

7. Define point of inflection.

8. Give any pair of isomorphic graphs.

9. $\lim_{x \to \infty} \left(\pi - \dfrac{2}{x^2} \right) = ...?$

10. Find the differential of the function $y = x^4 - 2x^3 + 8$.

11. Evaluate $\sum_{k=1}^{5} k^2$.

12. Evaluate the integral $\int_0^3 x^2 \, dx$.

(12 × 1 = 12 Marks)

Part B
Answer any **nine** questions. Each question carries 2 marks.

13. Evaluate the limit $\lim_{x \to -5} \dfrac{x^2 + 3x - 10}{x + 5}$.

14. Discuss the type of discontinuity of the function $y = sin\left(\dfrac{1}{x}\right)$ at $x = 0$,

15. State intermediate value theorem.

16. Where does the curve $y = \dfrac{x}{x - 1}$ have slope -1?

17. Suppose the dollar cost of producing x washing machines is $C(x) = 2000 + 100x - 0.1x^2$. Find the marginal cost when 100 washing machines are produced.

18. Find the critical points of the function $f(x) = -x^3 + 12x + 5$.

19. Evaluate the limit $\lim_{x\to-\infty} \dfrac{3x+7}{x^2-2}$.

20. Find the linearization of $f(x) = \sqrt{x^2+9}$ at $x = -4$.

21. Evaluate the limit $\lim_{x\to 0+}(x\ln x)$.

22. Express the sum $1 + \dfrac{3}{4} + \dfrac{5}{9} + \dfrac{7}{16} + \dfrac{9}{25}$ in sigma notation.

23. Find the average value of $f(x) = 3x^2 - 3$ on $[0, 1]$.

24. Evaluate the integral $\int_0^\pi (1 + \cos x)\, dx$.

(9 × 2 = 18 Marks)

Part C
Answer any *six* questions. Each question carries 5 marks.

25. Using $\varepsilon - \delta$ definition of limits prove that $\lim_{x\to x_0} k = k$.

26. Show that the function $f(x) = \left|\dfrac{x\sin x}{x^2+2}\right|$ is continuous at every value of x.

27. Write an equation for the tangent to the parabola $y = x - 2x^2$ at the point $(1, -1)$.

28. Find all derivatives of the function $y = 6t^4 - 3t^3 + 7t - 11$.

29. Verify mean value theorem for the function $f(x) = \log x$ on the interval $[1, e]$.

30. Determine the constants a and b so that the curve $y = x^3 + ax^2 + bx$ has an inflection at the point $(3, -9)$.

31. Evaluate the limit $\lim_{x\to 0+}(\cot x)^{\sin 2x}$.

32. Let $f(x) = x^3, 0 \le x \le 1$. Then prove that f is integrable over $[0, 1]$.

33. Find the area of the region enclosed by the parabola $x = y^2$ and the line $x = y + 2$.

(6 × 5 = 30 Marks)

Part D
Answer any *two* questions. Each question carries 10 marks.

34. (i) Evaluate the limit $\lim_{x\to 2-} \dfrac{x^2 - 3x + 2}{x^3 - 4x}$.
    (ii) Is there a real number that is one less than its 5th power?

35. Sketch the graph of $f(x) = \dfrac{(x+1)^2}{1+x^2}$.

36. Use definite integral to estimate the sum of the square roots of the first n positive integers,
    $\sqrt{1} + \sqrt{2} + ... + \sqrt{n}$.

(2 × 10 = 20 Marks)

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