

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS1 C01 / CC20U MTS1 C01 - MATHEMATICS - 1

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours

Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)Answer *all* questions. Each question carries 2 marks.

1. Find $\lim_{x \rightarrow 3} \frac{8x^2}{1 + \sqrt{x}}$
2. Find the slope of the tangent line to the graph of $y = x^4 - 2x^3 + 1$ at $x = 1$.
3. Find the slope of the tangent line to the graph of $f(x) = \frac{1}{\sqrt{x}}$ at $x = 2$.
4. Let $f(x) = \sqrt{x}$ and $g(x) = x^3 - 5$. Find $f \circ g$ and $g \circ f$.
5. If $x^4 + y^2 + y - 3 = 0$, what is $\frac{dy}{dx}$ when $x = 1, y = 1$?
6. Find $\int \frac{x^2 + 3}{\sqrt{x}} dx$
7. Let $f(x)$ be the step function defined by $f(x) = \begin{cases} -1 & \text{if } x < 0, \\ -2 & \text{if } x \geq 0, \end{cases}$
Show that f is discontinuous at x_0 .
8. Using algebra alone, show that $f(x) = x^2$ is increasing at $x_0 = -1$
9. Find the intervals on which $f(x) = 3x^3 - 8x + 12$ is concave upward and downward
10. State Horserace Theorem.
11. An object on the x axis has velocity $v = 2t - t^2$ at time t. If it starts out at $x = -1$ at time $t = 0$, where is it at time $t = 3$? How far has it travelled?
12. Calculate the integral $\int_2^3 \frac{dt}{t^2}$

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer *all* questions. Each question carries 5 marks.

13. Using the formal definition find the derivative of $f(x) = \frac{1}{x}$ at $x = -1$
14. Where does the line tangent to $y = \frac{x}{x+1}$ at $x = 1$ cross the x axis?
15. Find the equation of the line tangent to the parametric curve $x = t^2 + 1, y = \frac{1}{t^4 + 1}$ at $t = 2$.
16. Find the critical points, endpoints, maximum and minimum points and values of the function $f(x) = 4x^4 - 2x^2 + 1$ on the interval $[-10, 20]$
17. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x - x}{\tan x - x} \right)$
18. (a) Check the integral $\int x(1+x)^6 dx = \frac{1}{56}(7x-1)(1+x)^7 + C$
(b) Evaluate $\int_0^2 x(1+x)^6 dx$.
19. Find the area of the region between the graphs of x^2 and $x+3$ between $x = -1$ and $x = 1$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

20. (a) A Reservoir contains $10^8 - 10^4 t - 80t^2 - 10t^3 + 5t^5$ liters of water at time t , where t is the time in hours from when the gates are opened. How many liters per hour are leaving the reservoir after one hour?
(b) Find the velocity and acceleration of a moving particle at $t = 2$ if the position is given by $y = 18t^2 - 2t + 5$.
21. (a) Find the volume of the ball of radius r by using slice method.
(b) Find the average value of $x^2 \sin(x^3)$ on $[0, \pi]$

(1 × 10 = 10 Marks)
