21P103 (Pages: 2) Name: ....

Reg.No.: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C03 – REAL ANALYSIS - I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Let E be an infinite subset of a compact set K. Prove that E has a limit point in K.
2. Prove that continuous image of a compact metric space is compact.
3. Let f be real valued differentiable function on (a,b). If $f'(x) = 0\ \forall x \in (a,b)$, then prove that f is a constant.
4. Explain whether MVT is applicable to $f(x) = 2 + (x-1)^{\frac{2}{3}}$ in $[0,2]$.
5. If f is a real differentiable function defined on $[a,b]$ and $f'(a) < c < f'(b)$, prove that there is a point $x \in (a,b)$ such that $f'(x) = c$.
6. Let f be a bounded real valued function and α be a monotonic increasing function on $[a,b]$ such that $|f|$ is Riemann-Stieltjes integrable with respect to α. Is f Riemann-Stieltjes integrable with respect to α? Justify your answer.
7. When do we say that a curve is rectifiable? Let $\gamma : [0,1] \longrightarrow \mathbf{R^2}$ given by $\gamma(x) = (2x, x^2+1)$. Is γ rectifiable?
8. Let f be a bounded function and α be a monotonically increasing function on $[a,b]$. If the partition P' is a refinement of the partition P of $[a,b]$ then prove that $U(P',f,\alpha) \leq U(P,f,\alpha)$.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions in each unit. Each question carries 2 weightage.

Unit I

9. Prove that infinite subset of a countable set is countable.
10. If $K \subset Y \subset X$, then prove that K is compact relative to X if and only if K is compact relative to Y.
11. Prove that monotonic functions have no discontinuities of the second kind.

Unit II

12. State and prove Mean Value theorem for vector valued functions.
13. If $f \in \mathbf{R}(\alpha)$ on $[a,b]$, $m \leq f \leq M$, ϕ is continuous on $[m,M]$ and $h(x) = \phi(f(x))$ on $[a,b]$, prove that $h \in \mathbf{R}(\alpha)$ on $[a,b]$.

14. If f is bounded in $[a, b]$, f has only finitely many points of discontinuities on $[a, b]$ and α is continuous at every point at which f is discontinuous , prove that $f \in \mathbb{R}(\alpha)$ on $[a, b]$.

Unit III

15. Explain uniform convergence of series of functions. Show that the series $\sum_{n=1}^{\infty}(-1)^n(\frac{x^2+n}{n^2})$ converges uniformly in every bounded interval.

16. Let $\{f_n\}$ be a sequence of continuous functions defined on a set E, and if $f_n \longrightarrow f$ uniformly on E, then show that f is continuous on E.

17. Let $\mathbb{C}(X)$ denote set of all complex valued, continuous, bounded functions defined on the metric space X. Prove that $\mathbb{C}(X)$ is a complete metric space.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Prove that compact subsets of a metric spaces are closed.

    b) Let $f : X \longrightarrow Y$ be continuous where X and Y are metric spaces. If E is connected subset of X, prove that $f(E)$ is connected.

19. Show that there exist a real continuous function on the real line which is nowhere differentiable.

20. a) Let f be a bounded function on $[a, b]$. Prove the necessary and sufficient condition for f to be Riemann-Stieltjes integrable.

    b) Let f be a bounded function and α be monotonically increasing function on $[a, b]$. If $f_1 \in \mathbf{R}(\alpha)$, $f_2 \in \mathbf{R}(\alpha)$ on $[a, b]$, then prove that $(f_1 + f_2) \in \mathbf{R}(\alpha)$ on $[a, b]$.

21. If f is a continuous complex function on $[a, b]$, show that there exist a sequence of polynomials which converges uniformly to f on $[a, b]$.

(2 × 5 = 10 Weightage)

****