21P159 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C04 - PROBABILITY THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define minimal σ field. Prove that the intersection of arbitrary number of field is a σ field.
2. Define independence of events and classes.
3. Describe the properties of distribution function.
4. Prove that for any $(x_1, x_2) \in \mathbb{R}^2$, $F(x_1, x_2)$ is non-decreasing in each of its arguments.
5. Prove or disprove converse of multiplication theorem is not true.
6. Let X_n be a sequence of independent random variables with $P[X_n = e^n] = \frac{1}{n^2}, P[X_n = 0] = 1 - n^{-2}, n = 1, 2, \ldots$. Examine if the sequence converges almost surely to zero.
7. Define convergence in rth mean. If $X_n \underset{\rightarrow}{r} X$ then show that $E|X_n|^r \longrightarrow E|X|^r$.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. (a) Define conditional probability measure.
   (b) Define induced probability space.
9. State and prove C_r - inequality.
10. Prove that a sequence of independent random variables either converges almost surely or diverge almost surely.
11. Define convergence in probability. If $X_n \xrightarrow{P} X$ and $C \in \mathbb{R}$ is a constant, then show that $CX_n \xrightarrow{P} CX$.

12. Derive the integral characteristic function of distribution function.

13. State and prove Kolmogorov's three series theorem.

14. State and prove necessary and sufficient condition to hold WLLN's.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Define characteristic function. Check whether $|\phi(t)|$ is integrable in the following case, and if so obtain the probability density function using inversion theorem. $\phi(t) = e^{i2t}$.

16. (a) State and prove inversion theorem on characteristic functions.
    (b) State and prove uniqueness theorem of characteristic functions.

17. (a) If $X_n \xrightarrow{L} C$ where C is a constant, then show that $X_n \xrightarrow{P} C$.
    (b) Let $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{L} C$, where C is a constant, then show that $\frac{X_n}{Y_n} \xrightarrow{L} \frac{X}{C} (C \neq 0)$.

18. (a) State and prove Liapounov's central limit theorem.
    (b) State Lindeberg-Feller's central limit theorem.

(2 × 5 = 10 Weightage)

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