20U402 (Pages: 2) Name: ....

Reg.No: ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS4 C04 / CC20U MTS4 C04 - MATHEMATICS - 4

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** qestions. Each question carries 2 marks.

1. Verify that $x = ksin4t$, where k is an arbitrary constant, is a solution of the linear differential equation $\dfrac{d^2y}{dx^2} + 16x = 0.$
2. Verify that $y = \dfrac{1}{x^2+c}$ is a one-parameter family of solutions of the first order differential equation $y^{'} + 2xy^2 = 0$. Find a solution of the initial value problem $y^{'} + 2xy^2 = 0, y(2) = \dfrac{1}{3}$.
3. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{-x}{y}, y(4) = -3.$
4. Find the general solution of $\dfrac{dy}{dx} + 2y = 0.$
5. Define linear dependence and linear independence.
6. Solve $25x^2y^{''} + 25xy^{'} + y = 0.$
7. If $f(t) = (t+1)^3$, find $\mathscr{L}\{f(t)\}$
8. State first shifting theorem. Use it to evaluate $\mathscr{L}\left\{(1 - e^t + 3e^{-4t})\cos 4t\right\}$
9. Find the convolution $4t * 3t^2$
10. Prove that the product of two odd functions is even.
11. Check whether the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial x \partial y} - 3\dfrac{\partial^2 u}{\partial y^2} = 0$ is hyperbolic, parabolic or elliptic.
12. Write the one-dimensional heat equation.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** qestions. Each question carries 5 marks.

13. Solve $2xydx + (x^2 - 1)dy = 0.$
14. Solve $t^2\dfrac{dy}{dt} + y^2 = ty.$
15. Solve the initial value problem $y^{''} + y^{'} + 2y = 0, y(0) = 0, y^{'}(0) = 0.$
16. Find the general solution of $y^{''} + y = cos^2x$ by using Variation of Parameters.
17. Evaluate $\mathscr{L}^{-1}\left(\dfrac{2s-4}{(s^2+s)(s^2+1)}\right)$
18. Item If $f(t)$ is piecewise continuous on $[0, \infty)$ of exponential order and periodic with period *T*, prove that $\mathscr{L}\{f(t)\} = \dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st}f(t)dt$
19. Using Laplace transforms solve the initial value problem $y'' + 5y' + 4y = 0$ with $y(0) = 1$ and $y'(0) = 0$

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. a) Solve the boundary value problem $y^{''} + y = x^2 + 1, y(0) = 5, y(1) = 0.$

    b) Solve $y^{''} - 6y^{'} + 9y = 6x^2 + 2 - 12e^{3x}$
21. Find the Fourier series expansion of $f(x) = \begin{cases} 0, & \text{if} \quad -\dfrac{\pi}{2} < x < 0 \\ \cos x, & \text{if} \quad 0 \le x < \dfrac{\pi}{2} \end{cases}$

(1 × 10 = 10 Marks)

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