19U604 (Pages: 2) Name: ……..……..……….

Reg. No: ……….……….

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS-UG)

CC19U MTS6 B13 - DIFFERENTIAL EQUATIONS

(Mathematics - Core Course)

(2019 Admission - Regular)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Determine the order of the differential equation $(1 + y^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = e^x$. Also state whether the equation is linear or non – linear.
2. Write down the general form of a separable differential equation. And show that every separable equation is exact.
3. Does the differential equation $\frac{dy}{dt} = y$ has a solution passing through the point (1,0)?
4. Verify that the given functions are solutions of the differential equation $y'' - y = 0$

   a) $y_1 = e^t$ b) $y_2 = \cosh t$
5. Define the integrating factor of a differential equation. Show that μ (x) = x is an integrating factor of $(x^2 - 2x + 2y^2)\,dx + 2xy\,dy = 0$
6. Find wronskian of $y_1 = \sin t$ and $y_2 = \cos t$. Determine whether y_1 and y_2 are linearly independent.
7. Find a differential equation whose roots are e^{2x} and e^{3x}
8. Solve the homogeneous linear differential equation $y'' - 4y = 0$
9. Find a general solution for the equation $x^2y'' + 4xy' + 2y = 0,\ x > 0$
10. Use the method of variation of parameters, solve the differential equation $y'' + y = \sec x$
11. Define unit step function and write its Laplace transform
12. Find $L^{-1}\left(\frac{s^2-3s+4}{s^3}\right)$
13. Define fundamental period of a function. Find the fundamental period of sin5x.
14. State whether the function f(x) = xcosx is even or odd.
15. Show that the function defined by $u(x,y) = \ln(x^2+y^2)$ is a solution of the following partial differential equation. $u_{xx} + u_{yy} = 0$

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. Solve the initial value problem (y+2)dx + y(x+4)dy = 0; y(-3) = -1
17. Solve: $2x^2yy' = \tan(x^2y^2) - 2xy^2$
18. Make the following equation exact and hence solve ydx + (x^2y - x)dy = 0
19. Using the method of reduction of order solve the differential equation $t^2y''-5ty'+9y = 0$, t>0 , given that y = t^3 is a solution.
20. Prove that L(t^n) = $\frac{n!}{s^{n+1}}$
21. Find the inverse Laplace transform of the function f(t) = $\frac{3s+1}{(s-1)(s^2+1)}$
22. Using convolution find the inverse Laplace Transform of the function $\frac{1}{s(s^2+\omega^2)}$
23. Obtain the Fourier half range cosine series for the function $f(x) = x$ for $x \in [0, \pi]$.

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. Solve the differential equation $(2x - 4y + 5)y' + x - 2y + 3 = 0$
25. Solve $y'' + 2y' - 35y = 12e^{5t} + 37\, sin5t$
26. Solve using Laplace Transform: $y'' - 3y' + 2y = 4e2t, y(0) = -3, y'(0) = 5$
27. Find the Fourier series expansion of the function f(x) ,which is periodic with period 2π and which in $-\pi < x < \pi$ is given by f(x) = $\begin{cases} x, 0 \le x \le \pi \\ 2\pi - x, \pi \le x \le 2\pi \end{cases}$

(2 × 10 = 20 Marks)

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