19U602 (Pages: 2) Name: ………….……….

Reg. No…………..………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS-UG)

CC19U MTS6 B11 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2019 Admission - Regular)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 5

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Find the derivative of $(z^2 - (1 + i)z + 3)$.
2. (a) The period of e^z is …..

   (b) The period of e^{iz} is ……
3. Write the principal value of $Ln((1 + i)^4)$ in the form of $a + i\, b$.
4. Define a fundamental region.
5. State Cauchy-Goursat theorem.
6. Use Cauchy's integral formula to evaluate $\oint_C \frac{z^2-4z+4}{z+i}\, dz$ where C is the circle $|z| = 2$.
7. Find the maximum modulus of $f(z) = -iz + 1$ on the circular region $|z| \leq 5$
8. State the fundamental theorem for contour integrals.
9. Evaluate $\int_C \frac{e^z}{z - \pi i} dz$ where C is the circle $|z| = 4$.
10. Expand $e^{\frac{3}{z}}$ in a Laurent series for $0 < |z| < \infty$.
11. Determine the order of the poles of $f(z) = \frac{3z - 1}{z^2 + 2z + 5}$.
12. Identify the type of singularity of the function $\frac{\sin z}{z^2}$.
13. State the argument principle.
14. Find the residue at each pole of the function $f(z) = \frac{z}{z^2+16}$.
15. Use Cauchy's residue theorem to evaluate $\oint_C \frac{1}{(z-1)^2(z-5)}\, dz$ where the contour C is the circle $|z| = 2$.

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. Show that the function $f(z) = x^2 + y^2 + 2ixy$ is not analytic at any point but is differentiable along the x-axis.

17. Show that $w = e^z$ maps the fundamental region $-\infty < x < \infty$, $-\pi < y \leq \pi$ onto the set $|w| > 0$.

18. Evaluate $\oint_C \frac{z^2}{z^2+4}\, dz$ along (a) $|z - i| = 2$ and (b) $|z + 2i| = 1$ using Cauchy's integral formula.

19. Find an upperbound for the absolute value of $\oint_C \frac{e^z}{z+1}\, dz$ where C is the circle $|z| = 4$.

20. Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$ in a Laurent series valid for $0 < |z - 3| < 2$.

21. Determine whether the given sequence $\{\frac{3ni + 2}{n + ni}\}$ converges or diverges.

22. Use Rouche's theorem to show that none of the zeros of $g(z) = z^2 + 10z^3 + 14$ lie within the disk $|z| < 1$.

23. Find the residue at each pole of the given function $f(z) = \frac{5z^2 - 4z + 3}{(z+1)(z+2)(z+3)}$.

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. Find all solutions to the equation $\sin z = 5$.

25. Evaluate (a) $\int_C xy^2 dx$ (b) $\int_C xy^2 dy$ and (c) $\int_C xy^2 ds$ where the path of integration C is the quarter circle defined by $x = 4\cos t,\ y = 4\sin t,\ 0 \leq t \leq \frac{\pi}{2}$.

26. State and prove Taylor's theorem.

27. Evaluate the Cauchy principal value of $\int_0^{\infty} \frac{\sin x}{x^2 + 9}\, dx$.

(2 × 10 = 20 Marks)

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