21P201 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C06 - ALGEBRA- II

(Mathematics)

(2019 Admission onwards)

Time : 3 HoursMaximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that FF is a field, every proper non trivial prime ideal of F[x]F[x] is maximal.
2. Prove that R(i)≅CR(i)≅C
3. Show that regular 30-gon is constructible.
4. Prove that if EE is a field of characteristic pp then EE contains exactly $p_n$pn elements for some positive integer nn.
5. Prove that any two algebraic closures of a field FF are isomorphic.
6. Prove that CC is a splitting field over RR.
7. Give an example of a perfect field.
8. Prove that the polynomial x_n−axn−a is solvable by radicals over QQ.

(8 × 1 = 8 Weightag

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. A finite extension field EE of a field FF is an algebraic extension of FF . What about the converse?
10. Find a basis and dimension of for Q(3–√,7–√)Q(3,7) over QQ.
11. Let EE be an extension field of FF, then prove that F¯¯¯¯E={α∈E:α is algebraic over F}F¯E={α∈E:α is algebraic over F} is a subfield of EE.

UNIT - II

12. If FF is a field of prime characteristic pp with algebraic closure F¯¯¯¯F¯, then x_{p_n}−xxpn−x has $p_n$pn distinct zeros in F¯¯¯¯F¯.
13. Find the splitting field of x_3−2x3−2 over QQ.

14. If EE is a finite extension of FF, then show that EE is separable over FF if and only if each αα in EE i
separable over FF.

UNIT - III

15. Let KK be a finite normal extension of FF, and let EE be an extension of FF,
where F≤E≤K≤F¯¯¯¯.F≤E≤K≤F¯. Then show that

1. KK is a finite normal extension of EE
2. G(K/E)G(K/E) is precisely the subgroup of G(K/F)G(K/F) consisting of all those automorphisms that leave EE fixed.

16. If KKsplitting field of x_4+1x4+1 over QQ, prove that G(K/Q)G(K/Q) is isomorphic to Klein 4-group.

17. Find Φ_8(x)Φ8(x) over Z_2.Z2.

(6 × 2 = 12 Weightag

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let RR be a commutative ring with unity and MM. is an ideal in RR .Then prove that MM is a maximal ideal of RR if and only if R/MR/M is a field.

19. State and Prove Kroneckers Theorem.

20. State and Prove Isomorphism extension theorem.

21. Prove that the Galois group of pth cyclotomic extension of QQ for a prime pp is cyclic of order p−1.p−1.

(2 × 5 = 10 Weightag

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