**20P403** (Pages: 2) Name: ..............................

Reg. No.: .........................

**FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022**

(CBCSS - PG)

(Regular/Supplementary/Improvement)

**CC19P MTH4 E05 - ADVANCED COMPLEX ANALYSIS**

(Mathematics - Elective Course)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

## Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. When a set $\mathcal{F} \subseteq C(G, \Omega)$ is said to be normal?
2. What do you meant by $\mathcal{F} \subseteq H(G)$ is locally bounded?
3. Let $d$ be the metric in $\mathbb{C}_\infty$. Prove that $d\left(\frac{1}{z_1}, \frac{1}{z_2}\right) = d(z_1, z_2)$.
4. Show that the residue of the gamma function $\Gamma$ at simple pole $-n$ is given by $Res(\Gamma, -n) = \frac{(-1)^n}{n!}$
5. Define Riemann zeta function.
6. Prove that $\frac{1}{e^z - 1}$ has a pole of order one at $z = 0$.
7. Define analytic continuation along a path.
8. What do you meant by a finite genus entire function?

**($8 \times 1 = 8$ Weightage)**

## Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Prove that $C(G, \Omega)$ is a complete metric space.
10. State and prove Montel's theorem.
11. Let $Re(z_n) > 0, \forall n \in \mathbb{N}$. Prove that $\prod_{n=1}^{\infty} z_n$ converges to a nonzero number iff the series $\sum_{n=1}^{\infty} \log z_n$ converges.

UNIT II

12. Prove that $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$ if $Re(z) > 0$.
13. If $Re(z) > 1$, prove that $\zeta(z) = \prod_{n=1}^{\infty} \left(\frac{1}{1 - P_n^{-z}}\right)$ where $\{P_n\}$ is the sequence of prime numbers.
14. State and prove Runge's theorem.

Unit III

15. State and prove Mittag-Leffler theorem.

16. Suppose that G is a region, $G^* = \{z : \bar{z} \in G\}$, $G_+ = \{z \in G : Im(z) > 0\}$ and $G_0 = \{z \in G : Im(z) = 0\}$ such that $G = G^*$. If $f : G_+ \cup G_0 \to \mathbb{C}$ is a continuous function which is analytic on $G_+$ and if $f(x)$ is real for $x \in G_0$. Prove that there is an analytic function $g : G \to \mathbb{C}$ such that $g(z) = f(z)$ for all $z \in G_+ \cup G_0$

17. Derive the Jensen's formula.

**$(6 \times 2 = 12$ Weightage)**

## Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Suppose G is open in $\mathbb{C}$. Prove that there is a sequence $\{K_n\}$ of compact subsets of G such that $G = \cup_{n=1}^{\infty} K_n$ satisfying

    (a) $K_n \subseteq int\ K_{n+1}$

    (b) $K \subseteq G$ and $K$ compact implies $K \subseteq K_n$ for some $n$.

    (c) Every component of $\mathbb{C}_\infty - K_n$ contains a component of $\mathbb{C}_\infty - G$

19. Prove that a set $\mathcal{F} \subseteq C(G, \Omega)$ is normal iff the following two conditions are satisfied

    (a) for each $z \in G$, $\{f(z) : f \in \mathcal{F}\}$ has compact closure in $\Omega$.

    (b) $\mathcal{F}$ is equicontinuous at each point of G.

20. Show that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$ for all $z \in \mathbb{C}$. Using factorizations of $\sin \pi z$ find factorizations of $\cos \pi z$, $\sinh \pi z$ and $\cosh \pi z$

21. State and prove monodromy theorem.

**$(2 \times 5 = 10$ Weightage)**

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