20P404 (Pages: 2) Name: ……………………..…...

Reg. No……………………..

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH4 E09 - DIFFERENTIAL GEOMETRY

(Mathematics - Elective Course)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that graph of a function $f: \mathbb{R}^n \to \mathbb{R}$ is a level set of some function $F: \mathbb{R}^{n+1} \to \mathbb{R}$
2. Find and Sketch gradient field of $f(x_1, x_2) = x_1^2 + x_2^2$
3. Define spherical image of the oriented n-surface. Describe the spherical image of $x_2^2 = x_1$
4. Define Levi Civita parallelism. State any three properties
5. Show that normal component of acceleration is same for all parametrized curves in n-surface S in $\mathbb{R}^{n+1}$ passing through a point p with velocity **v**
6. Find length of the parametrized curve $\alpha: I \to \mathbb{R}^2\ where\ , \alpha(t) = (t^2, t^3), I = [0,2]$
7. Distinguish between first fundamental form and second fundamental form of an n-surface S
8. Let S be an n- surface which is the image of a parametrized n- surface $\varphi: U \to \mathbb{R}^{n+1}$with $N^{\varphi}(p) = N^S(\varphi(p)\ )$ for all $p \in U$. Show that $L_p^{\varphi} = L_{\varphi(p)}^S$

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Define Integral curve of a vector field. Find the integral curve through $p = (1,1)$ of the vector field $X(p) = -p$
10. State and prove Lagrange multiplier theorem
11. Define oriented n- surface Show that a connected n- surface in $\mathbb{R}^{n+1}$ has exactly two orientations. Give example of unorientable surface

UNIT II

12. Show that if $\alpha: I \to \mathbb{R}^{n+1}$ is a parametrized curve with constant speed then α is a geodesic. Check whether $\alpha(t) = (\cos 3t, \sin 3t)$ is a geodesic
13. Define Weingarten map. Prove that Weingarten map is self adjoint
14. Show that local parametrizations of the plane curve are unique upto reparametrization.

UNIT III

15. Let S be a compact connected oriented n- surface in$\mathbb{R}^{n+1}$ Then $K(p)$ of S is nonzero for all $p \in S$ if and only if second fundamental form of S at p is definite for all $p \in S$
16. Define differential of a smooth map.

    Let $\varphi: U \to \mathbb{R}^3$ be given by $\varphi(\theta, \phi) = (r\cos\theta\sin\phi\,, r\sin\theta\sin\phi\,, r\cos\phi\,)$ where $U = \{(\theta, \phi) \in \mathbb{R}^2 : 0 < \phi < \pi\}, r > 0$ Show that $d\phi$ is non singular for each $p \in U$
17. State and prove inverse function theorem for n- surfaces

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Show that Gauss map is onto.
19. Let S be an n- surface in $\mathbb{R}^{n+1}$,let $p \in S$ and let $\mathbf{v} \in S_p$ then there exist a maximal geodesic in S passing through p with initial velocity **v**
20. Let C be an oriented plane curve. Then prove that there exist a global parametrization of C if and only if C is connected
21. (a) Let S be an oriented n- surface in $\mathbb{R}^{n+1}$ and let $p \in S$. Let Z be non zero normal vector field on S. Derive Gauss Kronecker curvature formula for S at p

    (b) Find Gaussian Curvature of n- sphere of unit radius at (1,0…..,0)

(2 × 5 = 10 Weightage)

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