21U310S (Pages: 2) Name: …...……………………

Reg. No: ……….……………..

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CUCBCSS-UG)

CC15U MAT3 C03 / CC18U MAT3 C03 – MATHEMATICS - III

(Mathematics - Complementary Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark.

1. What is the order of the differential equation $y''' = e^x$.
2. Solve $y' - y = 0$.
3. Find the differential equation associated with the family of straight lines $x + y + c = 0$.
4. The rank of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ is …………
5. A consistent system of linear equations $AX = B$ in n unknowns have a unique solution if and only if ………….
6. The eigen values of a triangular matrix are …………… of the matrix.
7. Find a unit vector in the direction of the vector from (1,0,1) to the point (3,2,0).
8. Prove that $\boldsymbol{v} = 3y^4z^2\boldsymbol{i} + 4x^3z^3\boldsymbol{j} - 3x^2y^2\boldsymbol{k}$ is solenoidal.
9. Find gradient of the function $f(x, y) = x^2 - y^2$.
10. Give a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
11. Give an example of a non-orientable surface.
12. By Gauss divergence theorem $\iiint_T \; div\, \boldsymbol{F}\, dv =$………

(12 × 1 = 12 Marks)

PART B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Solve the differential equation $y' = (1 + x)(1 + y^2)$.
14. Show that $(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)dy = 0$ is exact.
15. Find an integrating factor of the differential equation $y' + y \tan x = \cos^3 x$.
16. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$.
17. Examine whether $x + 2y = 3$; $2x + 4y = 7$ form a consistent system of equations?
18. Find the angle between the lines $x - y = 1$ and $x - 2y = -1$.

19. Are the vectors $[1,2,1], [3,2,-7], [5,6,-5]$ linearly independent?
20. Find a unit vector perpendicular to both the vectors $\boldsymbol{a} = [4,-1,3]$ and $\boldsymbol{b} = [-2,1,-2]$.
21. If $\boldsymbol{v} = \sin t\ \boldsymbol{i} + \cos t\ \boldsymbol{j} + t\ \boldsymbol{k}$, evaluate $|\boldsymbol{v}'(t)|$.
22. Determine the unit tangent vector to the circle $x = \cos t,\ y = \sin t,\ z = 0$ at $t = 0$.
23. Find the magnitude of the greatest rate of change of $u = xyz^2$ at $(1,0,3)$.
24. Show that the line integral $\int_C (x^2y\,dx + 2xy^2dy)$ is path dependent in the xy-plane.

(9 × 2 = 18 Marks)

PART C

Answer any ***six*** questions. Each question carries 5 marks.

25. Solve the differential equation $(2x - 4y + 5)y' + x - 2y + 3 = 0$.
26. Find an integrating factor and hence solve the initial value problem

$$2\sin y^2\,dx + xy\cos y^2\,dy = 0$$

27. Using Crammer's rule solve $x + y + z = 3,\ \ x + 2y + 3z = 4,\ \ x + 4y + 9z = 6$.
28. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$.
29. A wheel is rotating about the x-axis with angular speed $3\,sec^{-1}$. The rotation appears clockwise if one looks from the origin in the positive x-direction. Find the velocity and the speed at the point $(2,2,2)$.
30. Find the volume of the tetrahedron with vertices $(1,1,1), (5,-7,3), (7,4,8)$ and $(10,7,4)$.
31. Find the arc length reparameterization of the helix $\boldsymbol{r}(t) = [a\cos t, a\sin t, ct]$.
32. Evaluate $\int_{(0,1,2)}^{(1,-1,7)} (3x^2dx + 2yzdy + y^2dz)$.
33. Using Stoke's theorem evaluate $\oint_C (xydx + xy^2dy)$, where C is the square in the xy-plane with vertices $(1,0), (-1,0), (0,1), (0,-1)$, oriented in the counterclockwise direction.

(6 × 5 = 30 Marks)

PART D

Answer any ***two*** questions. Each question carries 10 marks.

34. Find the orthogonal trajectories of the family of circles $x^2 + (y - c)^2 = c^2$.
35. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
36. Verify Green's theorem for the line integral $\oint_C (xydx + x^2dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

(2 × 10 = 20 Marks)

****