21U301 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 B03 / CC20U MTS3 B03 - CALCULUS OF SINGLE VARIABLE - II

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find $\int \frac{\sqrt{\ln x}}{x} dx$.
2. Find the derivative of $f(x) = \log_2 |\tan x|$.
3. Evaluate $\int \sqrt{\sinh x} \cosh x dx$.
4. Evaluate $\lim_{x \to 0} \frac{e^x - 1}{x^2 + x}$.
5. State root test for series.
6. Determine whether the series $\sum_{n=1}^{\infty} 5\left(\frac{4}{3}\right)^{n-1} = 5 + \frac{20}{3} + \frac{80}{9} + \frac{320}{27} + \cdots$ converges or diverges.
7. Determine whether the series $\sum_{n=1}^{\infty} n^{-1.001}$ converges or diverges.
8. Define conditionally convergent series.
9. Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{n!}{x^n}$.
10. Describe the curves represented by the parametric equations $x = 4cos\theta$ and $y = 3sin\theta$, with parameter interval $[0, 2\pi]$.
11. Sketch the region comprising points whose polar coordinates satisfy the condition $0 \leq r \leq 2$.
12. Sketch the cylinder $y = x^2 - 4$.
13. Find an equation in rectangular coordinates for the surface with spherical equation $\rho = 4cos\phi$.
14. Find the domain of the parameter t of the vector function $\overline{\gamma}(t) = \left\langle \frac{1}{t}, \sqrt{t-1}, \ln t \right\rangle$
15. If $x = \int_0^t \cos \frac{\pi u^2}{2} du$, $y = \int_o^t \sin \frac{\pi u^2}{2} du$ find $\frac{dy}{dx}$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the inverse of the function defined by $f(x) = \frac{1}{\sqrt{2x-3}}$.
17. Derive the formula for $\frac{d}{dx} \tan^{-1} x$.

18. Determine whether the series
$\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n-1}$ converges or diverges.

19. Determine whether the series
$\sum_{n=1}^{\infty} \frac{2^n}{n!\, n}$ absolutely convergent, conditionally convergent or divergent.

20. Find an equation of the tangent line to the curve $x = \theta cos\theta$, $y = \theta sin\theta$, at the point where $t = \frac{\pi}{2}$.

21. Find the length of the curve $r = 5sin\theta, 0 \leq \theta \leq 2\pi$.

22. Find an equation of the plane containing the point $(3, -3, 2)$ and having the normal vector $\mathbf{n} = \langle 4, 2, 3 \rangle$. Find the x-, y-, and z-intercepts, and make a sketch of the plane.

23. Find the parametric equations for the tangent line to the curve with parametric equations
$x = \sqrt{t+2}, y = \frac{1}{t+1}, z = \frac{2}{t^2+4}$ at $t = 2$.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Use implicit differentiation to find $\frac{dy}{dx}$ for $xe^{2y} - x^3 + 2y = 5$.

25. a) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2}$ converges or diverges.

b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{3+2^n}$ converges or diverges.

26. Find $\int e^{-x^2} dx$.

27. A shell is fired from a gun located on a hill $100m$ above a level terrain. The muzzle speed of the gun is $500m/sec$, and it's angle of elevation is $30°$.

a) Find the range of the shell.

b) What is the maximum height attained by shell?

c) What is the speed of the shell at impact?

(2 × 10 = 20 Marks)

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