21U302 (Pages: 2) Name: .....

Reg.No: .....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 C03 / CC20U MTS3 C03 - MATHEMATICS - III

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. If $\mathbf{r}(t) = \langle \ln t, 1 \rangle, t > 0$. Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.
2. Describe the level surfaces of the function $f(x, y, z) = \dfrac{x^2 + y^2}{z}$.
3. Find the level curve of $f(x, y) = \frac{x^2}{4} + \frac{y^2}{9}$ passing through the point $(-2, -3)$
4. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, prove that $\nabla \times \vec{r} = 0$
5. Show that $\int_{(0,0)}^{(1,1)} ydx + xdy$ is path independent.
6. State Green's theorem in the plane.
7. Find Jacobian of the transformation $x = \dfrac{y}{x^2}, y = \dfrac{y^2}{x}$ from uv-plane to the xy-plane.
8. Express $1 + i$ in polar form.
9. Differentiate $f(z) = \frac{3z-4+8i}{2z+i}$
10. Express $Ln(1 + i)$ in the form $a + ib$.
11. Evaluate $\oint_C \left(z + \dfrac{1}{z-4} \right) dz$, where C is the circle $|z| = 2$.
12. Evaluate $\int_{\frac{i}{2}}^{i} e^{\pi z} dz$

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Find the curvature of the elliptical helix given by $\mathbf{r}(t) = a\cos t\mathbf{i} + b\sin t\mathbf{j} + ct\mathbf{k}$, where $a, b, c > 0$.

14. Evaluate $\oint_C (x^2 - y^2)ds$ where C is given by $x = 5\cos t, y = 5\sin t, 0 \leq t \leq 2\pi$.

15. Find the surface area of the portions of the sphere $x^2 + y^2 + z^2 = a^2$ that are within the cylinder $x^2 + y^2 = b^2; \quad 0 < b < a$.

16. Convert the point $\left(\frac{1}{3}, \frac{5\pi}{3}, \frac{\pi}{6}\right)$ given in spherical coordinates to rectangular coordinates and cylindrical coordinates.

17. If $\mathbf{F} = xy\mathbf{i} + y^2z\mathbf{j} + z^3\mathbf{k}$, evaluate $\iint_S \mathbf{F}.\mathbf{n}dS$ where S is the unit cube defined by $0 \leq x \leq 1, \quad 0 \leq y \leq 1, \quad 0 \leq z \leq 1$.

18. Verify that the function $u(x, y) = \log(x^2 + y^2)$ is harmonic. Also find *v*, the harmonic conjugate of *u*.

19. Using ML-inequality find an upper bound for the absolute value of $\oint_C \frac{e^z}{z^2 + 1}dz$, where C is the circle $|z| = 5$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Find a vector that gives the direction in which the function $f(x, y) = \tan(x^2 + y^2)$ decreases most rapidly at the point $\left(\sqrt{\frac{\pi}{6}}, \sqrt{\frac{\pi}{6}}\right)$. Also find the minimum rate.

21. State Cauchy's integral formula. Using it evaluate $\oint_C \frac{z^2 + 3z + 2i}{z^2 + 3z - 4}dz$ where,
    a. C is the circle $|z + 5| = \frac{3}{2}$
    b. C is the circle $|z| = 2$

(1 × 10 = 10 Marks)

****