22U110S (Pages: 2) Name: ……

Reg. No: ……

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CUCBCSS-UG)

CC15U MAT1 C01 / CC18U MAT1 C01 – MATHEMATICS

(Mathematics Complementary course)

(2015 to 2018 Admissions - Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark

1. $\lim_{x\to 2}\frac{f(x)-5}{x-2}=10$ then $\lim_{x\to 2} f(x)=\ldots$
2. State Sandwich theorem.
3. A function f is continuous at an interior point $x=c$ of its domain if $\lim_{x\to c} f(x)=\cdots$
4. The slope of the curve $y=\frac{1}{x^2}$ at $x=2$ equals...
5. At any time t, the position of a moving particle along $s-$axis is $s=t^3-3t^2+5t-1$. What is its velocity when $t=5$.
6. Find the point of inflection of the curve $y=(x-2)^3+1$
7. How fast the area of a circle change with its radius, when radius is 5cm?
8. Find the second derivative of $f(x)=2x^5+7x^2-5x+3$.
9. Evaluate $\sum_{k=1}^{4}(-1)^{k+1}\sin k\pi$
10. Find the norm of the partition $\{0, 0.2, 0.45, 0.75, 1.5, 2.3, 3\}$
11. If $f(x)=-5$ on $[-2,-1]$, then evaluate $\int_{-2}^{-1} f(x)dx$.
12. Evaluate the integral $\int_0^{\pi} cos^2x\ dx$

(12 × 1 = 12 Marks)

PART B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Evaluate $\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x}$
14. Prove that $\lim_{x\to x_0} k=k$ by using the definition of limit.
15. Find the linearization of $f(x)=x^2-2x+5$ at $x=1$.
16. Find an equation for the tangent of the curve $y=(x-1)^2+1$ at the point (1,1).
17. Find absolute maximum and minimum values of $f(x)=x^3-3x$ on $[-2,5]$.
18. Show that $|x|$ is not differentiable at $x=0$.
19. Find the slope of tangent to the curve $k(x)=\frac{1}{2+x}$ at $x=-2$.
20. Verify Rolle's theorem for the function $f(x)=x^3-9x$ in the interval $[0,3]$.
21. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x+x^2}$.

22. If $r(x) = x^3 - 3x^2 + 12x$ gives the dollar revenue from selling x candies, $5 \leq x \leq 25$. What is the marginal revenue? What is the increase in revenue that will result from selling one additional unit if 12 candies are already sold?
23. Find the average value of $f(x) = 3x^2 - 3$ on $[0, 1]$.
24. Evaluate $\int_{-2}^{2} 3x^3\sqrt{x^4 + 1}\, dx$.

(9 × 2 = 18 Marks)

PART C

Answer any ***six*** questions. Each question carries 5 marks.

25. Is there a real number that is one less than its fifth power?
26. Find the equations of all lines having slope -1 that are tangent to the curve $y = \frac{1}{x-1}$.
27. Find intervals on which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is decreasing and increasing.
28. Find all derivatives of the function $y = x^4 - 4x^3 + 10x - 5$.
29. An object is dropped from the top of a $100\ m$ high tower. Its height above after t seconds is $(100 - 4.9t^2)m$. How fast is it falling 2 seconds after it is dropped?
30. Verify Mean Value theorem for the function $f(x) = x(x-1)(x-2)$ on $\left[0, \frac{1}{2}\right]$.
31. The diameter of a sphere is measured as 100 ± 1 cm and the volume is calculated from this measurement. Estimate the percentage error in the volume calculation.
32. Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ cannot possibly be 2.
33. Find the area of the region enclosed by the parabola x= y^2 and the line $x = y + 2$.

(6 × 5 = 30 Marks)

PART D

Answer any ***two*** questions. Each question carries 10 marks.

34. a) If $f(x) = 4 - x^2$, $x_0 = -1$, $\varepsilon = 0.5$ and $L = 3$, find $\delta > 0$ such that $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

    b) For what values of a, is $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ is continuous at every x?

35. a) For what values of a, is $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ is continuous at every x?

    b) Verify Mean Value Theorem for the function $f(x) = \log x$ on the interval $[1, e]$.

36. a) Using Riemann sum, evaluate $\int_a^b x\, dx$.

    b) The solid lies between planes perpendicular to the x- axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to x- axis are squares with sides run from the semicircle $y = -\sqrt{1 - x^2}$ to the semicircle $y = \sqrt{1 - x^2}$. Find the volume of the solid.

(2 × 10 = 20 Marks)

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