22U108 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS1 B01 / CC20U MTS1 B01 - BASIC LOGIC AND NUMBER THEORY

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Define paradox and give an example.
2. Write the inverse and contrapositive of a statement.
3. Verify that $\sim (p \wedge q) \equiv \sim p \vee \sim q$.
4. Define uniqueness quantifiers and give example.
5. Write (a) Addition law (b) Hypothetical syllogism.
6. Compute the first four terms of the sequence defined recursively $a_1 = 1, a_2 = 2, a_n = a_{n-1} + a_{n-2}$
7. Define transitive property of divisibility.
8. State the prime number theorem.
9. Find the five consecutive composite numbers less than 100.
10. State Lame's Theorem
11. State Dirichlet's Theorem
12. Solve if possible, Mahavira's puzzle if there were 24 travelers.
13. Prove or disprove $78 \equiv 48 (mod 5)$ can be reduced to $13 \equiv 8 (mod 5)$.
14. Using divisibility test determine whether 548 and 152 are divisible by 11.
15. Without using Wilsons theorem verify that $(p-1)! \equiv -1 (mod p)$ for $p = 3$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Prove that there is no positive integer between 0 and 1.

17. Prove that there are infinitude of primes

18. Using canonical decompositions, find the gcd of each pair; 72, 108.

19. Using the canonical decompositions of 1050 nd 2574, find their lcm.

20. Let p be a prime and a any integer such that p does not divide a. Then show that $a^{p-1} \equiv 1(modp)$.

21. Solve the linear congruence $26x \equiv 12(mod17)$.

22. State and prove Euler's theorem.

23. Using Euler's theorem find the remainder when 25^{2550} is divided by 18.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. 1) Explain
    a) Proof of contrapositive.
    b) Direct proof.
    c) Proof by cases.
    d) Constructive existence proof.
    e) Counter example method.

    2) Prove that $\sqrt{2}$ is an irrational number.

25. State and prove Fundamental Theorem of Arithmetic.

26. a) Show that $a \equiv b(modm)$ if and only if a and b leave the same remainder when divided by m.

    b) Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.

27. a) Using inverses, find the incongruent solution of $5x \equiv 3(mod6)$.

    b) Using congruences solve $15x + 21y = 3$

(2 × 10 = 20 Marks)

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