FIRST SEMESTER B.Sc DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS1 C01 / CC20U MTS1 C01 - MATHEMATICS - 1

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours   Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Find $\lim\limits_{x\to\infty} \dfrac{2x+1}{3x+1}$

2. Find the slope of the tangent line to the graph of $y = x^4 - x^2 + 3x$ at $x = 1$.

3. Let $f(x) = 4x^5 - 13x$ and $g(x) = x^3 + 2x - 1$. Find derivative of $\dfrac{g(x)}{f(x)}$

4. Verify chain rule for $f(u) = u^2$ and $g(x) = \sqrt{x}$.

5. If $x^3 + y^3 = xy$, compute $\dfrac{dx}{dy}$ in terms of x and y.

6. Find the general antiderivative of the function $f(x) = \dfrac{x+1}{x^3}$

7. Show that there is a number x_0 such that $x_0^5 - x_0 = 3$

8. Define critical point of a function.

9. Write the concavity of $f(x) = 4x^3$ at the points $x = 1$ and $x = 0$

10. State Mean Value Theorem

11. An object moving in a straight line has velocity $v = 6t^4 + 3t^2$ at time t. How far does the object travel between $t = 1$ and $t = 10$?

12. Verify the formula $\dfrac{d}{dx} \int_a^x f(s)ds = f(x)$ for $f(x) = x$.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer *all* questions. Each question carries 5 marks.

13. Use the formal definition to find the derivative of $f(x) = \dfrac{1}{x^2}$ for $x \neq 0$.

14. Find the equation of the line tangent to the graph of $y = \sqrt{x} + \dfrac{1}{2(x+1)}$ at $x = 1$.

15. Find the equation of the line tangent to the parametric curve $x = \sqrt{t^4 + 6t^2 + 8t}, \quad y = \dfrac{t^2 + 1}{\sqrt{t} - 1}$ at $t = 3$.

16. Find the critical points, endpoints, maximum and minimum points and values of the function $y = x^4 - x^2$ on the intreval $[-1, \infty)$

17. Evaluate $\lim\limits_{x \to \infty} \left(\dfrac{x^4 + \ln x}{3x^4 + 2x^2 + 1} \right)$

18. (a) Check the integral $\int x(1+x)^6 \, dt = \dfrac{1}{56}(7x - 1)(1 + x)^7 + C$

(b) Evaluate $\int_0^2 x(1+x)^6 \, dt$.

19. A parabolic arch with base 10 meters and height 12 meters is erected. How much area does it enclose?

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

20. (a) A Reservoir contains $10^8 - 10^4 t - 80t^2 - 10t^3 + 5t^5$ liters of water at time t, where t is the time in hours from when the gates are opened. How many liters per hour are leaving the reservoir after one hour?

(b) Find the velocity and acceleration of a moving particle at $t = 2$ if the position is given by $y = 18t^2 - 2t + 5$.

21. The region under the graph of x^2 on $0 \le x \le 1$ is revolved about the x axis. Using disk method find its volume.

(1 × 10 = 10 Marks)

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