20U503 (Pages: 2) Name: ....

Reg.No: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - UG)

CC20U MTS5 B07 - NUMERICAL ANALYSIS

(Mathematics - Core Course)

(2020 Admission - Regular)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Use algebraic manipulations to show that the function $g(x) = \left(\dfrac{3+x}{x^2+2}\right)^{1/2}$ has a fixed point at p precisely when $f(p) = 0$ where $f(x) = x^4 + 2x^2 - x - 3$

2. Find an approximate root of $f(x) = \cos x - x = 0$ using Newton's method by taking $p_0 = \dfrac{\pi}{4}$.

3. Using the numbers $x_0 = 2, x_1 = 2.75$ and $x_2 = 4$, find the second Lagrange interpolating polynomial for $f(x) = \dfrac{1}{x}$. Use this polynomial to approximate $f(3) = \dfrac{1}{3}$

4. Using Newton's divided difference formula construct an interpolating polynomials of degree three for the data given in the table,

x	-0.1	0.0	0.2	0.3
$f(x)$	5.30	2.00	3.19	1.00

5. Using the forward-difference formula approximate the derivative of $f(x) = e^x - 2x^2 + 3x - 1$ at $x_0 = 0$ by considering $h = 0.2$.

6. Given $f(x) = xe^x$. By taking $h = 0.1$ and using midpoint formula find an approximation to $f''(2.0)$ correct to four decimal places. Also determine the actual error occurred in the approximation.

7. Write the Trapezoidal rule formula to approximate $\int_a^b f(x)\,dx$. What is the error term?

8. Write the closed Newton-Cotes formula for $n = 2$. What is its error term?

9. Does the function $f(t, y) = 1 + t^2 y$ satisfy a Lipschitz condition on $D = \{(t, y) : 0 \le t \le 1; -\infty \le y \le \infty\}$?

10. Show that the initial value problem $y' = y\cos t,\ 0 \le t \le 1,\ y(0) = 1$ has a unique solution.

11. Use midpoint method to approximate $y(1)$ given $y' = te^{3t} - 2y,\ y(0) = 0$.

12. Write the difference equation of the Adams-Moulton three-step implicit method.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer *all* questions. Each question carries 5 marks.

13. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$. Use the Bisection method to determine an approximation to the root that is accurate to at least within 10^{-2}.

14. Use method of false position to find solution of $x^3 + 3x^2 - 1 = 0$ for $[-3, -2]$ accurate to within 10^{-3}.

15. Using Newton's backward-divided-difference formula evaluate $P_4(2.0)$ corresponding to the data given in the table,

x	1.0	1.3	1.6	1.9	2.2
$f(x)$	0.7652	0.6201	0.4554	0.2818	0.1104

16. Values for $f(x) = xe^x$ are given in the following table. Use all the applicable three-point formulas to approximate $f'(2.0)$.

x	1.8	1.9	2	2.1	2.2
$f(x)$	10.8893	12.7032	14.7781	17.149	19.855

Determine the actual error occurred in each case.

17. Compare the Trapezoidal rule and Simpson's rule approximations to $\int_0^2 x^2\, dx$. Determine the actual error of approximation.

18. Use Euler's method to approximate the solution of the initial value problem $y' = e^{t-y}$, $0 \leq t \leq 1$, $y(0) = 1$ with $h = 0.5$. Obtain the actual solution and compare the actual error at each step to the error bound.

19. Use modified Euler's method to approximate the solution of the initial value problem $y' = \sin t + e^{-t}$, $0 \leq t \leq 1$, $y(1) = 0$, with $h = 0.5$. Compare the results to the actual values.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

20. Use secant method to find solution of $x - \cos x = 0$ for $[0, \frac{\pi}{2}]$ accurate to within 10^{-4}.

21. Use Runge-Kutta method of order four to approximate the solution of the initial value problem $y' = (y/t) - (y/t)^2$, $1 \leq t \leq 2$, $y(1) = 1$, with $h = 0.5$.

(1 × 10 = 10 Marks)

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