22P101 (Pages: 2) Name: ……………………

Reg. No: ……………………

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C01 - ALGEBRA-I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART – A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define isometry of $\mathbb{R}^2$. Write an example.
2. Find all subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_4$ of order 4.
3. Let X be a G-set. Then show that G_x is a subgroup of G for each $x \in X$.
4. Prove that every group of order 159 is cyclic.
5. Find the center of $S_3 \times D_4$.
6. Find isomorphic refinements of the two series $\{0\} < 10\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 25\mathbb{Z} < \mathbb{Z}$.
7. Show that the group S_5 is not solvable.
8. Prove that $x^3 + 3x + 2$ is irreducible in $\mathbb{Z}_5[x]$.

(8 × 1 = 8 Weightage)

PART – B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT 1

9. Prove that if m divides the order of a finite abelian group G, then G has a subgroup of order m.
10. Show that if M is a maximal normal subgroup of G if and only if G/M is simple.
11. Show that if H and K are normal subgroups of a group G ,then $H \cap K$ is a normal subgroup of G.

UNIT 2

12. Prove that for a prime number p, every group G of order p^2 is abelian.
13. State and Prove Second Sylow theorem.
14. Prove that no group of order 36 is simple.

UNIT 3

15. Give the addition and multiplication tables for group algebra $\mathbb{Z}_2G$ where $G = \{e, a\}$ is cycle of order 2.
16. Show that $x^4 - 22x^2 + 1$ is irreducible over $\mathbb{Q}$.
17. Determine all groups of order 10 up to isomorphism.

(6 × 2 = 12 Weightage)

PART – C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) State and Prove Burnside's Formula

    (b) How many distinguishable necklaces (with no clasp) can be made using seven different colored beads of the same size.
19. (a) State and Prove Second isomorphism theorem.

    (b) If N is a normal subgroup of G, and if H is any subgroup of G, then show that $H \vee N = NH = HN$
20. (a) State and Prove Cauchy's Theorem.

    (b) Prove that every group of order 1645 is abelian and cyclic.
21. (a) State and Prove Division Algorithm for $F[x]$.

    (b) State and Prove Factor Theorem.

(2 × 5 = 10 Weightage)

****