22P102 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C02 - LINEAR ALGEBRA

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Let V be a vector space over the field F and α be any vector in V then prove that $0\alpha = 0$
2. Define coordinate matrix of α relative to the ordered basis $\mathcal{B}$.
3. Let F be a field and let T be the linear operator on F^2defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$. Then prove that T is non-singular and find T^{-1}.
4. Give a linear functional on $\mathbb{R}^3$.
5. Define hyperspace of a vector space V.
6. Define characteristic value and characteristic vector of a linear transformation T on a vector space V.
7. Let $(|)$ be the standard inner product on $\mathbb{R}^2$. Let $\alpha = (2, 4), \beta = (-2, 2)$. If γ is a vector such that $(\alpha|\gamma) = -2$ and $(\beta|\gamma) = 6$, find γ .
8. Give an orthonormal set in $\mathbb{R}^3$ with standard outer product.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Let S be a non-empty subset of a vector space V. Prove that the set of all linear combinations of vectors in S is the subspace spanned by S.
10. Let V be a bvector space which is psanned by a finite set of vectors $\beta_1, \beta_2, \ldots \beta_m$. Then prove that any independent set of vectors in V is finite and contains no more than m elements.
11. Find the range, rank, null space and nullity of zero transformation and the identity transformation on a finite dimensional space V

UNIT - II

12. Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_3, 2x_2 - x_3)$. IF $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$and $\mathcal{B}' = \{\beta_1, \beta_2\}$, where $\alpha_1 = (1, 0, -1), \alpha_2 = (2, 2, 2), \alpha_3 = (1, 0, 0), \beta_1 = (1, 0), \beta_2 = (0, 1)$. Find the matrix of T relative to the pair $\mathcal{B}, \mathcal{B}'$

13. Let F be a field and let f be the linear functional on F^2 defined by $f(x_1, x_2) = 3x_1 + 4x_2$. Let $T(x_1, x_2) = (x_1 - 2x_2, 2x_1 + x_2)$ and $g = T^t f$. Find $g(x_1, x_2)$

14. Define T conductor of α into W. Prove that $S(\alpha, W)$ is an ideal in the polynomial algebra$F[x]$.

UNIT - III

15. Let V be a finite dimensional vector space. Let $W_1, W_2, \ldots W_k$be subspaces of V and let $W = W_1 + W_2 + \ldots + W_k$. Then prove that $W_1, W_2, \ldots W_k$are independent if and only if For each $j, 2 \leq j \leq k$,we have $W_j \cap (W_1 + W_2 + \cdots + W_{j-1}) = \{0\}$

16. If V is an inner product space then for any vectors $\alpha$and β in V and any scalr c prove that $||\alpha + \beta|| \leq ||\alpha|| + ||\beta||$, $||c\alpha|| = |c|||\alpha||$ and $||\alpha|| > 0$ for $\alpha \neq 0$.

17. Apply the Gram-Schmidt process to the vectors $\beta_1 = (3, 0, 4), \beta_2 = (-1, 0, 7), \beta_3 = (2, 9, 11)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. If W_1 and W_2 are finite dimensional subspace of a vector space V then prove that $W_1 + W_2$ is finite dimensional. Also verify $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

19. (a) Let T be a linear transformation from V in to W, where V and $W$are finite dimensional and $\dim V = \dim W$ then show that T is non-singular if and only if T is onto

    (b) Prove that every n dimensional vector space over the field F is isomorphic to the space F^n.

20. (a) Let T be a linear operator on an n dimensional vector space V. Show that the characteristic and the minimal polynomial for T have the same roots, except for multiplicities.

    (b) Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Find the characteristic and minimal polynomial for T.

21. (a) Prove that the mapping $\beta \to \beta - E\beta$ is the orthogonal projection of V on $W^\perp$. where V is an inner product space, W a finite dimensional subspace, and E the orthogonal projection of V on W.

    (b) State and Prove Bessel's Inequality.

(2 × 5 = 10 Weightage)

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