37. (i) The rate at which the volume of sales(Q) for a new type of printer increase after an advertising campaign is given by the equation $\dfrac{dQ}{dt} = 0.05(500 - Q)$, given that Q=0 at t=0. Q is the number of printers sold, t is the time in years. Solve the differential equation to obtain an expression for Q in terms of t.

(ii) Write down differential equations of the type limited and unlimited growth..

**(2 × 4 = 8 Weightage)**

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**Part A**

Answer *all* questions. Each question carries 1/5 weightage.

1. A negatively sloped curve moves:

    (a) Upward  (b) Downward  (c) Vertically  (d) Horizontally

2. A diagonal matrix in which each of the diagonal element's unity is called:

    (a) Unit matrix  (b) Triangular matrix  (c) Diagonal matrix  (d) Zero matrix

3. The value of the determinant
$\begin{vmatrix} a - b & a + b \\ a + b & a - b \end{vmatrix}$ is:

    (a) $-4ab$  (b) $4ab$  (c) $a^2 - b^2$  (d) None of these

4. The amount of fertilizer applied (x) and yield per plot (y) of a crop are related by an equation $y = 20.38 + 2.5x - 0.125x^2$. Then the quantity of fertilizer to be used to get maximum yield is equal to:

    (a) 20  (b) 10  (c) 20.38  (d) 2.5

5. The rank of the matrix $\begin{bmatrix} 6 & 4 \\ 12 & 8 \end{bmatrix}$ is:

    (a) 0  (b) 1  (c) 2  (d) None of these

6. $\lim\limits_{x \to 1} (x^3 + 4)$ is:

    (a) 5  (b) 1  (c) 7  (d) None of these

7. If $y = x \log x,$ then $\dfrac{dy}{dx}$ is equal to:

    (a) $1 + \dfrac{1}{x}$  (b) $1 + \log x$  (c) $1 + \frac{1}{x}$  (d) $x + \frac{1}{x}$

8. The slope of the supply function S = 2 + 4P is:

    (a) 0  (b) 2  (c) 4  (d) 7

9. The demand for a commodity is D= -3-p . The supply function is S = -9 +p , then the equilibrium price is:

    (a) 1  (b) 2  (c) 3  (d) 5

10. If $u = e^{-3xyz}$, then $\dfrac{\partial u}{\partial y}$ at x = 1, z = 1 is:

 (a) $-3e^{-3y}$  (b) $3e^{-4y}$  (c) $e^{-3y}$  (d) $-3e^{-3xyz}$

11. Which of the following approximates the marginal impact on the objective function caused by a small change in the constant of the constraint:

 (a) the Legrange multiplier  (b) the Jacobian

 (c) the Hessian  (d) the determinant

12. The producers surplus when the supply function is p = 10+2q and the equilibrium price 20 is:

 (a) 35  (b) 25  (c) 100  (d) 50

13. The order of the differential equation $\sqrt{1 + \dfrac{d^2y}{dx^2}}$ is:

 (a) First  (b) Second  (c) Third  (d) None of these

14. Which of the following areas can the difference equation be applied:

 (a) Cob-Web model  (b) Harrod Domar model

 (c) Both a and b  (d) None of these

15. At the rate of 8.5% p.a. simple interest, a sum of Rs.4800 will earn how much interest in 2 years 3 months ?

 (a) 796  (b) 816  (c) 918  (d) 956

**(15 × 1/5 = 3 Weightage)**

**Part B** (Very Short Answer Questions)

Answer any *five* questions. Each question carries 1 weightage.

16. Write any four properties of determinations.

17. If $y = (1 - \sqrt{x})(1 + \sqrt{x})$, find $\dfrac{dy}{dx}$.

18. Define price elasticity of demand.

19. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $z = \dfrac{x}{y}$.

20. The marginal cost function for a certain product is $MC = 3q^2 - 4q + 5$. Find the total cost function given the fixed cost is 100.

21. Solve $\dfrac{dy}{dx} + \dfrac{2}{x}y = \dfrac{1}{x}$.

22. The salary of employee increases every year by 7% of his initial salary and his initial basic salary is Rs.5000. Find his salary at the end of 10 th year.

23. Find the rate of interest per annum if the simple interest on a Principal of Rs. 6,000 is 800 for 6 years.

**(5 × 1 = 5 Weightage)**

**Part C** (Short Answer Questions)

Answer any *seven* questions. Each question carries 2 weightage.

24. Explain implicit and explicit functions with examples.

25. Obtain the inverse of matrix
$$\begin{bmatrix} 2 & -3 & 0 \\ 3 & 1 & -2 \\ -1 & 0 & -4 \end{bmatrix}$$

26. Solve the system of equations by Crammer's rule.
$$2x - 3y + 5z = 11, 5x + 2y - 7z = -12, -4x + 3y + z = 5.$$

27. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \\ 2 & 4 & 6 \end{pmatrix}$.

28. Define the term limit of a function. Find $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$.

29. The demand function faced by a firm is $p = 500 - 0.2x$ and its cost function is $C = 25x + 10000$ (p=price, x=output, C=cost). Find the output at which the profits of the firm are maximum. Also find the price it will charge.

30. Given: $Z = x^4 e^{3y}$. Find all partial derivatives of second order.

31. Optimize $TC = 35 + 5Q - 2Q^2 + 2Q^3$.

32. For the data given below determine (i) market price $P_t$ in any time period (ii) the equilibrium price $P_e$.
$Q_{dt} = 180 - 0.75\,P_t$, $Q_{st} = -30 - 0.3\,P_{t-1}$, $P_0 = 220$.

33. Find the amount at the end of 5 th year for Rs. 5000 at 10% p.a., simple interest. What is the total amount of growth?

**(7 × 2 = 14 Weightage)**

**Part D** (Essay questions)

Answer any *two* questions. Each question carries 4 weightage.

34. Find the adjoint of the matrix and verify that $A(\text{Adj}A) = |A|I$ if $A = \begin{pmatrix} 1 & 4 & 5 \\ 3 & 2 & 2 \\ 0 & 1 & -3 \end{pmatrix}$.

35. A firm has the following total cost and demand functions:
$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$ and $Q = 100 - p$. Find profit maximizing level of output; also find profit at this level of output.

36. Demand and Supply laws under pure competition are given by $p_d = 16 - x^2$ and $p_s = 4 + x$. Determine market price, consumer's surplus and producer's surplus.