22P107 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P PHY1 C02 - MATHEMATICAL PHYSICS - I

(Physics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Section A

Answer ***all*** questions. Each question carries 1 weightage.

1. What are the characteristics of orthogonal curvilinear coordinates?
2. Express Laplacian operator in cylindrical coordinates
3. Show that eigen values of a Hermitian matrix are real and eigen vectors belonging to different eigen values are orthogonal.
4. What do you mean by symmetric tensors and anti-symmetric tensors?
5. Define Hermitian operator. Write any two properties of Hermitian operator.
6. Define Legendre's Polynomial and Show that $P_0(x) = 1$
7. Show that Fourier series for an odd function consists of sine terms alone.
8. Define Fourier transform of a function.

(8 × 1 = 8 Weightage)

Section B

Answer any ***two*** questions. Each question carries 5 weightage.

9. Derive the expression for gradient,divergence and curl in general curvilinear co-ordinate system.Use the result to find the expressions for the same in spherical polar co-ordinates.
10. Obtain the series solution of Bessel's differential equation. Explain the limitation of the method.
11. Derive Trigonometric Expansion Involving Bessel Function. Prove That $J_n(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x}{2}(z-\frac{1}{z})}$
12. (a) Derive the generating function of Hermite Polynomial.
    (b) Derive Rodrigues formula of Hermite Polynomial.

(2 × 5 = 10 Weightage)

Section C

Answer any ***four*** questions. Each question carries 3 weightage.

13. A rigid body is rotating about a fixed axis with a constant angular velocity $\vec{\omega}$. Take ω to lie along the z axis. Express $\vec{r}$ in circular cylindrical coordinates and using circular cylindrical coordinates calculate
a) $\vec{v} = \vec{\omega} \times \vec{r}$ b) $\nabla \times \vec{v}$ c) $\nabla . \vec{r}$ d) $\nabla \times \vec{r}$

14. Using Gram-Schmidt orthogonalisation process, form an orthonormal set from the set of functions $u_n(x) = x^n$, n = 0, 1, 2, … in the interval $-1 \leq x \leq 1$ with the density functions w(x) =1.

15. Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

16. Prove that $\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin x\pi}$

17. Show That $\int_0^\infty \sin^P\theta \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{P+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{P+q+2}{2}\right)}$.

Hence evaluate $\int_0^\infty \sin^P\theta \, d\theta$ and $\int_0^\infty \cos^q\theta \, d\theta$.

18. Obtain Fourier sine and cosine integrals.

19. Using partial fraction expansion, find $L^{-1}\left[\frac{s+1}{s^2(s^2+a)}\right]$

(4 × 3 = 12 Weightage)

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