22U201 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS2 B02 / CC20U MTS2 B02 - CALCULUS OF SINGLE VARIABLE - I

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find the function of the transformed graph if
   (a) $f(x) = x^2 + x - 1$: shifted vertically upward by 3 units.
   (b) $f(x) = x^2 - 4$: shifted horizontally to left by 2 units.
2. Evaluate $lim_{x\to 0} \frac{\tan 2x}{3x}$.
3. What is a jump discontinuity? Give an example for it.
4. Find the rate of change of $y = \sqrt{2x}$ with respect to x at $x = 2$.
5. Find the rate of change of $y = 2x^3 + 2$ with respect to x at $x = 2$.
6. The position of a particle moving along a straight line is given by $s(t) = \frac{t}{t+1}, t \geq 0$ where t is measured in seconds and s in feet. Find the position, velocity and speed of the particle at $t = 0$
7. The total cost incurred in operating an oil tanker on an $800mi$ run, traveling at an average speed of ν mph, is estimated to be $C(\nu) = \frac{1{,}000{,}000}{\nu} + 200\nu^2$ dollars. Find the approximate change in the total operating cost if the average speed is increased from 10 mph to 10.5 mph
8. Find the linearization of $f(x) = \sqrt{2x+3}$ at $a = 3$
9. Using Mean value theorem verify the function f(x) = sin x ; [0, $\pi/2$] and find c.
10. Find the interval on which f(x) = x sin x + cos x, 0 < x < 2π is increasing or decreasing.
11. Define limit of a function at infinity.
12. Define horizontal asymptote.
13. A car is moving along a straight road with velocity function $V(t) = 2t^2 + t - 6; 0 \leq t \leq 8$, where $V(t)$ is measured in feet per second. Find the displacement of the car between t = 0 and t = 3.
14. Define a smooth function and a smooth curve.

15. Find the work done by a variable force $F(x) = \frac{1}{x^2}$N along the x-axis from $x = 1$ m to $x = 10$m

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Let $f(x) = \frac{1}{x+1}$ and$g(x) = \frac{x}{x+1}$.Find $f + g, f - g, fg, f/g.$

17. Let $\lim_{x\to 3} 2x^2 = 18$ and $\epsilon = 0.1$ .Find a number δ >0 such that $|f(x) - 18| < \epsilon$ whenever $0 < |x - 3| < \delta.$

18. (a) Explain the Extreme value theorem

    (b) Describe a procedure for finding the extrema of a continuous function f on a closed interval [a,b].

19. Using the definition of area, find the area of the region under the graph of $f(x) = 2x + 1$on [0, 2] by choosing C_k as the left end point.

20. Compute the Riemann sum for $f(x) = 4 - x^2$ on $[-1, 3]$ using the five subintervals (n = 5) and choosing the evaluvation points to be the mid point of the subintervals.

21. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on $[0, 2]$ about the x- axis

22. Find the area of the surface obtained by revolving the graph of $y = x^{1/3}$on the interval $[1, 8]$ about the y -axis

23. Find the center of mass of a system comprising three particles with masses 2, 4, and 1 grams, located at the points (-2,2),(2,1) and (3,-1) respectively. (Assume that all distances are measured in centimeters)

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) Discuss on Second derivative test.

    (b) State and prove the second derivative test.

    (c) Find the relative extrema of $f(x) = x^3 - 3x^2 - 24x + 32$ using the second derivative test.

25. Sketch the graph of the function $f(x) = 2x^3 - 3x^2 - 12x + 12.$

26. State and prove both Part 1 and Part 2 of Fundamental Theorem of Calculus.

27. Find the area of the region bounded by the graphs of $y = 2x + 4$ and $y = x^3$ and the horizontal line $x = 0$ using integration (i) with respect to x (ii) with respect to y

(2 × 10 = 20 Marks)

****