21U402 (Pages: 2) Name: .....

Reg.No: .....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS4 C04 / CC20U MTS4 C04 - MATHEMATICS - IV

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Write the difference between explicit and implicit solutions.
2. Verify that $y = \dfrac{1}{(1+ce^{-x})}$ is a one-parameter family of solutions of the first order differential equation $y' = y - y^2$. Find a solution of the initial value problem $y' = y - y^2, y(0) = \dfrac{-1}{3}$.
3. Define separable equations and give an example.
4. Define Bernoulli's equation.
5. Define n^{th} - order homogeneous and non homogeneous differential equation.
6. Define elastic curve.
7. Find $\mathscr{L}\{f(t)\}$, if $f(t) = \begin{cases} 0 & 0 \le t < 3 \\ 2 & t \ge 3 \end{cases}$
8. Evaluate $\mathscr{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\}$
9. Evaluate $\mathscr{L}\left(4t * 3t^2\right)$
10. Check whether the functions $f_1(x) = x^2$ and $f_2(x) = x^3$ defined on the interval $[-1, 1]$ are orthogonal or not.
11. Check whether the partial differential equation $3\dfrac{\partial^2 u}{\partial x^2} + 5\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = 0$ is hyperbolic, parabolic or elliptic.
12. Write the one-dimensional wave equation.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Solve the general solution of $(x^2-9)\dfrac{dy}{dx}+xy=0$.

14. Solve $(2y^2+3x)dx+(2xy)dy=0$.

15. Solve the initial value problem $y^{''}-4y^{'}-5y=0, y(1)=0, y^{'}(1)=2$.

16. Solve the initial value problem $x^2y^{''}+xy^{'}+y=0, y(1)=1, y^{'}(1)=2$.

17. Evaluate $\mathscr{L}^{-1}\left(\dfrac{2s-4}{(s^2+s)(s^2+1)}\right)$

18. If $f(t)$ is piecewise continuous on $[0,\infty)$ of exponential order and periodic with period *T*, prove that $\mathscr{L}\{f(t)\}=\dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st}f(t)dt$

19. Expand the function $f(x)=\begin{cases}\pi, & \text{if} \quad -1<x<0\\ -\pi, & \text{if} \quad 0\le x<1\end{cases}$ in an appropriate cosine or sine series.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. a) Solve the initial value problem $y^{''}+y=4x+10sinx, y(\Pi)=0, y^{'}(\Pi)=2$.
    b) Solve $y^{''}-6y^{'}+9y=6x^2+2-12e^{3x}$.

21. Using Laplace transforms solve the initial value problem $y''+y=4\delta(t-2\pi)$ with $y(0)=1$ and $y'(0)=0$ where $\delta(t-2\pi)$ is the Dirac delta function.

(1 × 10 = 10 Marks)

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