20U601 (Pages: 2) Name: …………..……….

Reg. No………….………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B10 / CC20U MTS6 B10 - REAL ANALYSIS

(Mathematics - Core Course)

(2019, 2020 Admissions)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 5

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Give an example for a continuous function on a set which has neither an absolute maximum nor an absolute minimum on the set.
2. State non uniform continuity criteria.
3. Show that $f(x) = \sin x$ is uniformly continuous on $[0, \infty)$.
4. Show that the equation $x = \cos x$ has a solution in $[0, \pi/2]$.
5. Prove that a constant function is Riemann integrable.
6. Suppose that $f \in \mathcal{R}[a, b]$. Prove that $kf \in \mathcal{R}[a, b]$ where $k \in \mathbb{R}$.
7. Give an antiderivative of the Signum function in $[-5, 8]$.
8. Evaluate $\int_0^2 t^2 \sqrt{1 + t^3}\, dt$ and justify your steps.
9. Show that $f_n(x) = \frac{x}{n};\ x \in \mathbb{R}, n = 1, 2, \cdots$ is not uniformly convergent on $\mathbb{R}$.
10. Find uniform norm of a real valued function.
11. State Weierstrass M- test for series of functions.
12. Examine the convergence of $\int_1^\infty \frac{\ln x}{x^2}\, dx$
13. Prove that Beta function is symmetric.
14. Find $\Gamma(\frac{1}{2})$
15. Find $\int_0^{\frac{\pi}{2}} sin^{5/2} x\ cos^{3/2} x\, dx$

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. State and prove Boundedness theorem of continuous functions.
17. Prove that, if $f; [0,1] \to [0,1]$ is a continuous function then $f(x) = x$ for atleast one x in $[0, 1]$.

18. State and prove Squeeze Theorem.
19. Prove that every continuous function is Riemann integrable.
20. Test the uniform convergence of the sequence $\{ e^{-nx} \}$ for $x \geq 0$.
21. Prove that if (f_n) is a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly to a function $f: A \to \mathbb{R}$, then f is continuous on A.
22. Prove that if $p, q > 0$, $B(p,q) = B(p+1,q) + B(p,\ q+1)$
23. Check the convergence of the improper integral $\int_0^\infty \frac{sin^2x}{x^2}\, dx$

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. State and prove Location of Roots theorem and deduce Bolzano's intermediate value theorem.
25. State and prove Additivity theorem.
26. Let (f_n) be a sequence of functions on $\mathcal{R}[a,b]$ and suppose that (f_n) converges uniformly on $[a,b]$ to f. Then prove that $f \in \mathcal{R}[a,b]$.
27. (a) Find the relation connecting Beta function and Gamma function.

    (b) Show that even though the improper integral $\int_{-1}^{5} \frac{dx}{(x-1)^3}$ does not converge, its Cauchy Principal Value exists.

(2 × 10 = 20 Marks)

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