20U603 (Pages: 2) Name: ....

Reg.No: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B12 / CC20U MTS6 B12 - CALCULUS OF MULTI VARIABLE

(Mathematics - Core Course)

(2019, 2020 Admissions)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Sketch a contour map for the surface described by $f(x,y) = x^2 + y^2$, using the level curves corresponding to $k = 0, 1, 4, 9$ and 16.
2. Find f_{yx} and f_{yy} if $f(x,y) = 2xy^2 - 3x^2 + xy^3$.
3. Find $\dfrac{dy}{dx}$ if $x^3 - 2xy + y^3 = 4$.
4. Find the gradient of $f(x,y) = x\sin y + y\cos x$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.
5. Let $f(x,y) = 2x^2 + y^2 - 6x + 2y + 1$. Find the critical point of f.
6. Evaluate$\int_0^1 \int_0^2 (x+2y)dydx$
7. Evaluate $\int\int_R (x+2y)dydx$, where $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le x\}$.
8. Evaluate $\int_0^2 \int_0^1 \int_0^3 xyzdzdydx$.
9. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^3 \int_0^r rdzdrd\theta$
10. Find the jacobian of the transformations T defined by the equations $x = u^2 - v^2, y = 2uv$.
11. Find the gradient vector field of $f(x,y) = e^{-2x}\sin 3y$.
12. Determine whether the vector field $F(x,y) = (2x^2 + 4y)\hat{\imath} + (2x - 3y^2)\hat{\jmath}$ is conservative.
13. Give the statement of Green's Theorem.
14. Find a parametric equation of the plane $2x + 3y + z = 6$.
15. Give the statement of Stockes' Theorem.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the area of that part of the plane $y + z = 2$ inside the cylinder.

17. Suppose z is a differentiable function of x and y that is defined implicitly by $x^2 + y^3 - z + 2yz^2 = 5$. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

18. Find equations of the tangent plane and normal line to the surface with equation $2x^2 - y^2 + 3z^2 = 2$ at the point $(2, -3, 1)$.

19. Find the volume of the solid that lies below the paraboloid $z = 4 - x^2 - y^2$, above the xy-plane and inside the cylinder $(x-1)^2 + y^2 = 1$.

20. Show that $lim_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{2x^2 + y^2}$ does not exist.

21. Show that the divergence of the electric field $E(x,y,z) = \dfrac{kQ}{|r|^3} r$, where $r = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, is zero.

22. Evaluate $\int_C ydx + zdy + xdz$, where C consist of part of the twisted cubic C_1 with parametric equations $x = t, y = t^2, z = t^3$, where $0 \leq t \leq 1$, followed by the line segment C_2 from $(1,1,1)$ to $(0,1,0)$.

23. Find the mass of the surface S composed of the part of the paraboloid $y = x^2 + z^2$ between the planes $y = 1$ and $y = 4$ if the density at a point P on S is inversely proportional to the distance between P and the axis of symmetry of S.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (i) A storage tank has the shape of a right circular cylinder. Suppose that the radius and height of the tank are measured at 1.5 ft and 5 ft, respectively, with a possible error of 0.5 ft and 0.1 ft respectively. Use differentials to estimate the maximum error in calculating the capacity of the tank.
(ii) Show that the function f defined by $f(x,y) = 2x^2 - xy$ is differentiable in the plane.

25. Find the maximum and minimum values of the function $f(x,y,z) = 3x + 2y + 4z$ subject to the constraints $x - y + 2z = 1$ and $x^2 + y^2 = 4$.

26. A lamina occupies a region R in the $xy-$ plane bounded by the parabola $y = x^2$ and the line $y = 1$. Find the center of mass of the lamina if its mass density at a point (x, y) is directly propotional to the distance between the point and the x-axis.

27. Let T be the region bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $z = 0$ and $x = 0$, and $x + z = 2$, and let S be the surface of T. If $F(x,y,z) = xy^2\hat{\imath} + \left(\frac{1}{3}y^3 - \cos xz\right)\hat{\jmath} + xe^y\hat{k}$, using Divergence Theorem find $\int\int_S F \cdot ndS$.

(2 × 10 = 20 Marks)

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