SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023
(CBCSS - UG)
(Regular/Supplementary/Improvement)
CC19U MTS6 B13 / CC20U MTS6 B13 - DIFFERENTIAL EQUATIONS
(Mathematics - Core Course)
(2019, 2020 Admissions)

Time : 2.5 Hours   Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Solve the initial value problem $\dfrac{dy}{dt} = -2y + 5;\ y(0) = 0$

2. Determine the order of the given differential equation $(1 + y^2)\dfrac{d^2 y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$ also state whether the equation is linear or nonlinear.

3. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{3x^2 + 4x + 2}{2(y - 1)},\ \ y(0) = -1$

4. Find an interval in which the initial value problem $ty' + 2y = 4t^2\ ; y(1) = 2$ has a unique solution.

5. Solve the linear differential equation $y'' + 7y' + 12y = 0$

6. Find the Wronskian of $y_1 = \sin t, y_2 = \cos t$. Determine whether y_1 and y_2 are linearly independent.

7. Find the general solution of $4y'' + 5y' - y = 0$

8. Find the general solution of $16y'' + 24y' + 9y = 0$

9. What is the radius of convergence of the Taylor series for $(1 + x^2)^{-1}$ about $x = 0$?

10. Find the Laplace transform of $\sin at$

11. Find the inverse Laplace transform of $\dfrac{b}{(s - a)^2 + b^2}$

12. Write the expression for $\mathcal{L}(f'(t))$ and $\mathcal{L}(f''t))$

13. Find the inverse Laplace transform of $G(s) = \dfrac{1}{(s + 2)^2 + 1}$

14. Determine whether the function $f(x) = |x|^3$ is even, odd, or neither.

15. Write heat equation and wave equation

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer *all* questions. Each question carries 5 marks.

16. Determine whether the equation $(2x + 3) + (2y - 2)y' = 0$ is exact. If it is exact, find the solution.

17. Find an integrating factor of $(3x^2y + 2xy + y^3) + (x^2 + y^2)y' = 0$

18. Find the particular solution of $y'' + 2y' + y = 3e^{-t}$

19. Use the method of variation of parameters find the general solution of the differential equation $y'' + 4y = 8\tan t \quad -\pi/2 < t < \pi/2$

20. Find the inverse Laplace transform of $\dfrac{2(s-1)e^{-2s}}{s^2 - 2s + 2}$

21. Find the Laplace transform of $f(t) = \int_0^t (t - \tau)^2 \cos(2\tau)d\tau$

22. Solve the boundary value problem $y'' + y = 0; y(0) = 1, y(\pi) = a$

23. Find the Fourier series of $f(x) = x, -1 \le x < 1; f(x + 2) = f(x)$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any *two* questions. Each question carries 10 marks.

24. Solve the initial value problem $y' + \dfrac{1}{4}y = 3 + 2\cos(2t); \ y(0) = 0$

25. Find the general solution of $y'' + 2y' = 3 + 4\sin(2t)$

26. Using Laplace transform, find the solution of the initial value problem
$y'' + \omega^2 y = g(t); \quad y(0) = 0, y'(0) = -1$

27. Let $f(x) = \begin{cases} x & -\pi \le x < 0, \\ 0, & 0 \le x < \pi, \end{cases}$ and suppose that $f(x + 2\pi) = f(x)$. Find the Fourier series for f.

(2 × 10 = 20 Marks)

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