22U302 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 C03 / CC20U MTS3 C03 - MATHEMATICS - III

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. If $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + \tan^{-1} t\mathbf{k}$. Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.
2. Find the gradient of the function $f(x, y) = y - e^{-2x^2 y}$
3. Find the level curve of $f(x, y) = -x^2 + y^2$ passing through the point (2,3). Also find the gradient at the point.
4. Find the divergence of the vector field $\vec{F}(x, y, z) = (x - y)^3\vec{i} + e^{-yz}\vec{j} + xye^{2y}\vec{k}$
5. Determine whether the vector field $\mathbf{F} = (4x^3y^3 + 3)\mathbf{i} + (3x^4y^2 + 1)\mathbf{j}$ is conservative.
6. State Stokes' theorem.
7. Convert the equation $z = 2r\sin\theta$ to rectangular coordinates.
8. Express $5 - 5i$ in polar form.
9. Sketch the graph of $Im(z) = -2$.
10. Show that $i^{2i} = e^{-(1+4n)\pi}$
11. Evaluate $\oint_C \dfrac{dz}{z^2}$, where C is the ellipse $\dfrac{(x-2)^2}{1} + \dfrac{(y-5)^2}{4} = 1$.
12. Evaluate $\displaystyle\int_{-1}^{-1+i} 2z\,dz$

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. If $z = e^{uv^2}$ and $u = x^3, v = x - y^2$. Using chain rule find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$

14. Evaluate $\oint_C (x^2 - y^2)ds$ where C is given by $x = 5\cos t, y = 5\sin t, 0 \leq t \leq 2\pi$.

15. Using Green's theorem find the work done by the force $\mathbf{F} = (-16y + \sin x^2)\mathbf{i} + (4e^y + 3x^2)\mathbf{j}$ acting along the positively oriented simple closed curve C which is the boundary of the region in the upper half plane bounded by the graphs of $y = x, y = -x$ and $x^2 + y^2 = 1$.

16. If $\mathbf{F} = xy\mathbf{i} + y^2z\mathbf{j} + z^3\mathbf{k}$, evaluate $\iint_S \mathbf{F}.\mathbf{n}dS$ where S is the unit cube defined by $0 \leq x \leq 1, \quad 0 \leq y \leq 1, \quad 0 \leq z \leq 1.$

17. Show that the Jacobian of the transformation from spherical to rectangular coordinates $\dfrac{\partial(x, y, z)}{\partial(\rho, \phi, \theta} = \rho^2 \sin\phi.$

18. Verify that the function $u(x, y) = e^x(x\cos y - y\sin y)$ is harmonic. Also find *v*, the harmonic conjugate of *u*.

19. Using ML-inequality find an upper bound for the absolute value of $\oint_C \dfrac{e^z}{z^2+1}dz$, where C is the circle $|z| = 5$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. The position of a moving particle is given by $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 3t\mathbf{k}$. Find the vectors $\mathbf{T}, \mathbf{N}$ and $\mathbf{B}$. Also find the curvature.

21. State Cauchy's integral formula. Using it evaluate.
    a. $\oint_C \dfrac{(z-1)}{z(z-i)(z-3i)}dz$ where C is the circle $|z - i| = \frac{1}{2}$
    b. $\oint_C \dfrac{\sin z}{z^2 + \pi^2}dz$ where C is the circle $|z - 2i| = 2$

(1 × 10 = 10 Marks)

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