23P155 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22P MST1 C02 - ANALYTICAL TOOLS FOR STATISTICS – II

(Statistics)

(2022 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. State and prove basis theorem.
2. If W_1 and W_2 are finite subspace of a vectorspace V, then prove that
   $d(W_1 + W_2) = d(W_1) + d(W_2) - d(W_1 \cap W_2)$.
3. Explain (i) symmetric matrix (ii) Idempotent matrix and (iii) Nilpotent matrix with an example of each.
4. Explain computation of inverse of a matrix by partitioning.
5. Illustrate the reduction of symmetric matrix to a diagonal form.
6. State and prove the necessary and sufficient condition for a real quadratic form X'AX to be negative definite.
7. Define quadratic forms. Show that the form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$ is an indefinite quadratic form.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Write a short note on Gram- Schmidt orthogonalization process.
9. If A and B are idempotent matrices, then show that the rank of idempotent matrix is equal to its trace.
10. Explain elementary operations of a matrix. Reduce the following matrix in to row reduced echelon form.
    $$A = \begin{bmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & 13 \end{bmatrix}$$
11. If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the characteristic roots of a matrix A, Show that trace (A) $= \sum_{i=1}^{n} \lambda_i$
12. Illustrate the reduction of a symmetric matrix to a diagonal form.
13. Define reflexive g-inverse. Show that a reflexive g-inverse always exist and it is not unique.

14. Find rank, index and signature of the real quadratic form $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (a) Define Kernel and image of linear transformation.

(b) Let T: $R^3 \rightarrow R^4$ be a linear map defined by T $(x_1, x_2, x_3) = (x_1, x_1 + x_2, x_1 + x_2 + x_3, x_3)$. Find Ker T and Im T.

(c) Consider the mapping f : $R^4 \rightarrow R^3$, given by f (a,b,c,d) = (a+b, b+c,a+d). Find Ker T and Im T.

16. (a) State and prove Cayley- Hamilton theorem.

(b)Verify Cayley-Hamilton theorem for A= $\begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and hence find A^{-1}.

17. (a) Define singular value decomposition.

(b) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigen values of a square matrix A not necessarily distinct then prove that product of eigen values equal to determinant of A.

18. (a) State and prove necessary and sufficient condition for the diagonalizability of a square matrix.

(b) Prove that If A and B be two symmetric matrices such that the roots of the equation $|A - \lambda B| = 0$ are all distinct then there exist a matrix P such that P^TAP and P^TBP are both diagonal matrices.

(2 × 5 = 10 Weightage)

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