23P157 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22P MST1 C04 - PROBABILITY THEORY

(Statistics)

(2022 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define monotone field. Show that a sigma field is monotone field and conversely.

2. Let $(\Omega, \mathscr{F}, P)$ be the probability space and $\{A_n , n\geq 1 \}$ be a sequence of events in $\mathscr{F}$, If $A_n \longrightarrow A$, then show that P(An) $\longrightarrow$ P(A) in the case of monotone increasing sequence of events.

3. Test whether the following is a distribution function.
   i) $F(x) = tan^{-1}x; -\infty < x < \infty$ ii)$F(x) = \frac{2}{\pi}tan^{-1}x; 0 < x < \infty$

4. Define Mathematical expectation of a random variable X. Examine whether E(X) exists when x follows the probability density function $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$

5. Define convergence in rth mean. Examine the convergence in rth mean for the sequence of random variables $\{X_n, n \geq 1\}$ with $P[X_n = n] = \frac{1}{n}, P[X_n = 0] = 1 - \frac{2}{n}$ and $P[X_n = -n] = \frac{1}{n}, n = 1, 2, \ldots$

6. Define complete convergence of a seqence of distribution function $\{F_n, n \geq 1\}$. If $F_n(x) = \begin{cases} 0, \, if\, x < 0 \\ 1 - e^{-nx}, \, if\, x \geq 0 \end{cases}$ examine whether it is completely convergent or not.

7. Let $X_1, X_2, \ldots, X_n$ be identically and independently distributed random variables according to exponential distribution with density $f(x) = \theta e^{-\theta x}, x > 0$. Define $S_n = X_1 + X_2 + \ldots + X_n$ Show that $Z_n = \frac{S_n - \frac{n}{\theta}}{\frac{\sqrt{n}}{\theta}}$ follows standard normal distribution when $n \longrightarrow \infty$

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. If A and B are two independent events defined over a probability space $(\Omega, \mathscr{A}, P)$ then prove the following.
   i) A and B^c are independent
   ii) A^c and B are independent
   iii) $A^c \; and \; B^c$ are independent

9. State and prove Basic inequality.

10. a) Derive the inversion formula for derive the probability mass function of an integer valued random variable.

b) If the characteristic function of a random variable X is $\phi_x(t) = (q + pe^{it})^n$ derive its probability mass function

11. Define tail sigma field. Prove that tail event has probability either zero or one.

12. Define convergence in probability. If $X_n \xrightarrow{P} X$ and $C \in \mathbb{R}$ is a constant, then show that $CX_n \xrightarrow{P} CX$.

13. State and prove Levy's continuity theorem for characteristic function.

14. State and prove Kolmogorov inequality

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Derive the characteristic function of a random variable X having probability density function as follows

i) $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$

ii) $f(x) = \begin{cases} 1+x, & \text{if } -1 \leq x < 0 \\ 1-x, & \text{if } 0 < x \leq 1 \end{cases}$

16. a) State and prove Taylor series expansion on characteristic function.

b) The characteristic function of an integer valued random variable is $\phi_x(t) = \frac{p}{1-qe^{it}}$. Derive the probability mass function of X using inversion formula

17. a) Show that convergence on probability implies convergence in distribution.

b) Let $\{F_n(x), n \geq 1\}$ be a sequence of distribution functions defined by

$F_n(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1 - \frac{1}{n}, & \text{if } 0 \leq x < n \\ 1 & \text{if } x \geq n \end{cases}$ Examine whether the sequence $\{F_n(x), n \geq 1\}$ converges in distribution.

18. a) Let $\{X_k\}, k = 1, 2, 3, \ldots$ be a sequence of independent random variables where each variable of the sequence X_k takes values k *and* $-k$ with equal probabilities. Examine whether WLLN holds or not

b) State and prove Lindeberge- Levy's central limit theorem.

(2 × 5 = 10 Weightage)

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