23P102 (Pages: 2) Name: ....

Reg. No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C02 - LINEAR ALGEBRA

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find all the subspaces of $\mathbb{R}^2$
2. Define coordinate matrix of α relative to the ordered basis $\mathcal{B}$.
3. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by $T(x_1, x_2) = (\sin x_1, x_2)$. Check whether T is linear or not.
4. Let $\mathcal{B} = \{\alpha_1, \alpha_2\}$ be the basis for $\mathbb{R}^2$ defined by $\alpha_1 = (3, 5)$ and $\alpha_2 = (1, 4)$. Find the dual basis of $\mathcal{B}$.
5. Let F be a field and let f be the linear functional on F^2 defined by $f(x_1, x_2) = ax_1 + bx_2$. Let $T(x_1, x_2) = (-x_2, x_1)$ and $g = T^t f$. Find $g(x_1, x_2)$.
6. Define an invariant subspace of a vector space V. Let T be any linear operator on V then prove that rang of T is invariant under T
7. If V is an inner product space, the for any vectors α, β in V and any scalar c prove that $||c\alpha|| = |c|||\alpha||$
8. Give an orthogonal set in $\mathbb{R}^3$ with standard inner product.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Show that the n tuple space F^n is a vector space.
10. Let V be a finite- dimensional vector space $n = \dim V$. Then prove that any subset of V which contains more than n vectors is linearly independent.
11. Find the inverse of the linear transformation $T(x_1, x_2) = (x_2, x_1 + 4x_2)$.

UNIT - II

12. Let V be a finite dimensional vector space over the field F. For each vector α in V define $L_\alpha(f) = f(\alpha),\ f \in V^*$. Then prove that the mapping $\alpha \to L_\alpha$ is an isomorphism of V onto V^{**}.

13. Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_2 - x_3)$. IF $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ and $\mathcal{B}' = \{\beta_1, \beta_2\}$, where $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (1, 0, 0), \beta_1 = (0, 1), \beta_2 = (1, 0)$. Find the matrix of T relative to the pair $\mathcal{B}, \mathcal{B}'$.

14. Let T be a linear operator on the dinite dimensional space V Let $c_1, c_2, \ldots, c_k$ be the distict characteristic values of T and let W_i be the space of characteristic vectors associated with the characteristic value c_i If $W = W_1 + W_2 + \ldots + W_k$, then show that if $\mathcal{B}_i$ is an ordered basis for W_i, then $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2 \ldots, \mathcal{B}_k)$ is an ordered basis for W.

UNIT - III

15. Define projection on a vector space V. Prove that

(a) Any projection E is diagonalizable.

(b) If E is projection on R along N, then $(I - E)$ is the projection on N along$R$.

16. Define an inner product on the space $F^{n \times n}$, the space of all $n \times n$ matrices over F.

17. Apply the Gram-Schmidt process to the vectors $\beta_1 = (3, 0, 4), \beta_2 = (-1, 0, 7), \beta_3 = (2, 9, 11)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) If $W_1$and $W_2$are finite - dimensional subspaces of a vector space V then prove that $W_1 + W_2$ is finite- dimensional also prove that $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$

(b) If $W$is a proper subspace of a finite dimensional space V, the prove that $W$is finite dimensional and $\dim W < \dim V$

19. Let V be an n dimensional vector space over the field F, prove that $L(V, V)$ is finite dimensional and has dimension n^2.

20. Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. If $V$and W are finite dimensional then prove the following.

(a) $\text{rank}(T^t) = \text{rank}(T)$

(b) The range of T^t is the annihilator of the null space of T^t.

21. (a) Let V be an inner product space, W a finite dimensional subspace, and E the orthogonal projection of V on W. Prove that the mapping $\beta \to \beta - E\beta$ is the orthogonal projection of V on $W^\perp$.

(b) State and Prove Bessel's Inequality

(2 × 5 = 10 Weightage)

****