23P104 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C04 - DISCRETE MATHEMATICS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that even when a maximal element is unique it need not be a maximum element.
2. Define distributive lattice. Also justify the statement that there exist lattice which are not distributive with an example.
3. Defiine symmetric boolean function with proper explanation.
4. Define self complementarity of a simple graph G with an example.
5. Define vertex cut with an example.
6. If the grith k of a connected plae graph G is at least 3, then $m \leq \frac{k(n-2)}{(k-2)}$.
7. Find the grammer $L = \{a^n b^{n-3} : n \geq 3\}$.
8. Define extented transition function.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. If x, y are elements of a boolean algebra. prove that $x = y$ if and only if $xy' + x'y = 0$.
10. State and prove Stone representaton theorem for finite boolean algebra.
11. Write the DNF of $g(a, b, c) = (a + b + c)(a' + b + c')(a + b' + c')(a' + b' + c')(a + b + c')$

UNIT - II

12. A simple graph is a tree if and only if any two distinct vertices are connected by a unique path.
13. Prove that $K_{3,3}$ is nonplanar.

14. (a) Let G be a graph and f be a face of G. Then there exists a plane embeddng of G in which f is the exterior face.

(b) Let G be a planar graph. Then G an be embedded in the plane in such a way that any specified vertex (or edge) belongs to the unbounded face of the resulting plane graph.

UNIT - III

15. Prove if u and v are strings then the length of their concatenation is the sum of individual length.

16. Consider the NFA with final state is q_1 and draw the transition graph with $\delta(q_0, a) = q_1, \delta(q_1, \lambda) = q_2, \delta(q_2, \lambda) = q_0$. Find $\delta(q_1, a), \delta^*(q_1, a), \delta^*(q_2, \lambda), \delta(q_2, aa)$.

17. Contruct a DFA equivalent to the given NFA, $\delta(q_0, a) = q_1, \delta(q_1, a) = q_1, \delta(q_1, \lambda) = q_2, \delta(q_2, b) = q_0$. where q_1 is the final state.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let X be a boolean algebra, then

(a) Find out all the boolean functions in two variables.

(b) Find out all the atoms of this boolean algebra.

(c) List all the symmetric boolean functions from the above collection.

(d) Find out the characteristic numbers of all symmetric boolean functions listed above.

19. The connectivity and edge connectivity of a simple cubic graph G are equal.

20. State and prove Whitney's theorem on 2- connected graphs.

21. (a) Consider the grammer $G = (\{s\}, \{a, b\}, S, P)$, where P is given by $S \to asb, s \to \lambda$. Then $s \Rightarrow asb \Rightarrow a(asa)b \Rightarrow aabb$, therefore $s \Rightarrow^* aabb$. Therefore the string $aabb$ is a sentence in the language generated by G, while $aasbb$ is a sentential form. In fact $L(G) = \{a^n b^n : n \geq 0\}$.

(b) Show that $|u^n| = n|u|$ for all strings u and for all n.

(2 × 5 = 10 Weightage)

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