PHY2C06:MATHEMATICAL PHYSICS – II

A Part

1. Evaluate the differentiability of the function $f(z) = \frac{1}{z}$
2. Show that the function $x^2 + iy^3$ is not analytic anywhere.
3. Show that the real and imaginary parts of complex analytic function satisfy Laplace equation.
4. Explain the concept of contour integral.
5. State Cauchy's theorem.
6. Give a brief explanation of Laurent series.
7. Briefly explain the difference between Taylor expansion and Laurent expansion.
8. Locate and name all the singularities of $f(z) = \frac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$
9. Explain different types of singularities.
10. Find the residue of the function $f(z) = \frac{z}{(z-1)(z+1)^2}$ at z=-1.
11. Explain the Cauchy principle value of integrals.
12. Define a group. Give an example.
13. What is a symmetry transformation? What is the order of the group of symmetry transformations of a square?
14. State and prove the Rearrangement theorem.
15. Find the identity element and inverse of the following groups: a) Set of all real numbers under addition b) The group {1,i,-i, -1} under multiplication
16. Discuss the conditions that define a group.
17. Generate a group starting from an element A subject to the condition $A^5 = E$, where E is the identity element. Identify the inverse of each element of the group also.
18. What is an Abelian group? Give an example.
19. State and explain rearrangement theorem
20. Define classes of a group. Prove that all elements of a class have the same order.
21. If a group H of order h is a subgroup of another group G of order G, show that H and the coset XH are disjoint sets if X is not in H.
22. Check whether the group {i,-1,-i,1} is cyclic?
23. Write a note on conjugate elements and classes.
24. What is a cyclic group? Is a cyclic group Abelian or non-Abelian?
25. State and prove Lagrange theorem of subgroups
26. Distinguish between isomorphism and homomorphism.

27. Write a short note on Cosets and define a normal subgroup
28. Explain direct product of groups.
29. State and prove Lagrange theorem of subgroups
30. Define a subgroup. What is the difference between a subgroup and a proper subgroup?
31. Can a group of order five have a subgroup of order three? Support your argument with necessary theory .
32. Define a normal subgroup. Write one normal subgroup of the group C_{4v}.
33. Find the subgroups of ={E,A,B,C} , if AB = BA = C, AC = CA = B and BC = CB = A.
34. Write any three proper subgroups of the group C_{4v}.
35. Prove Lagrange's theorem regarding the order of a group and that of its subgroup.
36. Show that the identity element of a group is a class by itself.
37. Define a factor group. What is its order?
38. What is meant by reducible representation.
39. Distinguish between reducibility and irreducibility.
40. Write a note on permutation groups
41. Distinguish between reducible and irreducible representations.
42. What is an alternating group? What is the order of an alternating group?
43. What is meant by irreducible representation.
44. Write a note on Lie groups.
45. Give the generators of SU(3) group.
46. What are the features of an SO(2) group.
47. Write a note on SU(3) eightfold way
48. Discuss any two forms of Euler equation and its applications.
49. Name any three applications of Euler equations.
50. Explain the concept of variation.
51. Describe the significance of calculus of variation methods
52. Discuss about the calculus of variation technique.
53. What idea does the Euler equation convey if x does not appear explicitly in the integrand i.e.,$f = f(y, y_x)$.
54. Give one application of Euler equation
55. Using Euler equation, determine the shortest distance between two point in euclidean x-y plane
56. Using Euler equation, prove that the shortest distance between two point in euclidean x-y plane represents a straight line.

57. Write down the Euler equation and name any two applications.
58. What are Euler equations in calculus of variation. Write down its alternate forms.
59. Discuss the Euler equation and its different forms. Name any two applications of Euler Equation.
60. Give the Euler equation for several independent variables
61. Write the expression for Euler equation in the case of several independent variables
62. Discuss the advantage of Hamilton Principle- Lagrangian formulation.
63. Obtain the equation that forms the basis of the Rayleigh -Ritz method for the computation of eigen functions and eigen values.
64. Describe the significance of Lagrangian Multiplier's in calculus of variation.
65. Give the expression for euler equation in the case of several dependent and independent variables
66. Obtain the expression for Euler equation in the case of several dependent and independent variables
67. Give the Euler equation for several dependent variables
68. Write the expression for Euler equation in the case of several dependent variables
69. What is Lagrangian multiplier in calculus of variation?
70. Write the expression for Euler equation in the presence of constraints.
71. Obtain the equation that forms the basis of the Rayleigh -Ritz method for the computation of eigen functions and eigen values.
72. Explain Rayleigh-Ritz variational technique.
73. Differentiate between Fredholm and Volterra integral equations
74. Describe the advantages of integral equations over differential equations.
75. Discuss the classification of integral equations based on the limits of integration involved
76. What are the different types of integral equations.
77. Give the general form of Fredholm and Volterra Equations. Describe the need for integral equations
78. Write down the general Volterra equations of the first kind and second kind.
79. Discuss the need for integral equations
80. Discuss the advantages of integral equations over differential equations.
81. How does the Fredholm integral equations differ from Volterra integral equations
82. Explain the difference between Fredholm and Volterra integral equations.
83. Define kernel of an integral equation and hence describe homogeneous and non homogenoeus integral equations.
84. Briefly explain the classification of integral equations based on the limits of integration involved
85. What are the advantages of integral equations over differential equations.

86. How can we differentiate between integral equations of first and second kind. Give a suitable example .
87. Write down the general Fredholm equations of the first kind and second kind.
88. Distinguish between integral equations of first and second kind with a suitable example .
89. Differentiate between homogeneous and non homogenoeus integral equations.
90. Discuss briefly about Fourier transform solution for a Fredholm equation of first kind.
91. What are the integral transform methods used in finding the solution for integral equations.
92. Name any two integral transforms (with relevant expression) that can be used in integral transform technique to find solution for integral equations
93. Briefly explain the use of generating function in the context of integral equation using the example of Legendre polynomials.
94. Discuss the various methods used to find the solution of integral equations
95. What are the different methods used to find the solution of integral equations
96. Briefly explain Seperable kernel method.
97. Briefly describe the Neumann Series method.
98. Discuss about Seperable kernel method.
99. Describe any two methods that are employed in solving integral equations.
100. What is the restriction in using Seperable Kernel method. Name any two other methods employed in finding the solution of an integral equation.
101. Write a short note on seperable Kernel method in solving integral equation.
102. Deduce the Neumann Series method.
103. Differentiate between Neumann Series method and Seperable Kernel method
104. Distinguish between Neumann Series method and Seperable Kernel method
105. Describe how the symmetry property of Green's function is connected with the eigen function expansion form of Green's function.
106. Give the eigen function expansion form of Green's function and hence explain the symmetry property
107. Briefly discuss the properties of Green function.
108. Explain how Green function is used in solving differential equation.
109. Write a brief note on Sturm-Liouville equation
110. Write a short note on Green's function and its properties.
111. Discuss the continuous behavior of Green's function and its derivative.
112. Obtain a general formula to calculate Green's function
113. What is the physical interpretation of the symmetry property of Green's function.

114. What is meant by Sturm -Liouville equation. Name any two properties of one dimentional Greens's function

115. Discuss the properties of one dimentional Greens's function

116. Describe the physical interpretation of the symmetry property of Green's function.

117. Deduce the relation between Dirac delta function and Green's function

118. Describe the eigen value-eigen function equation and expalin its significance based on Green's function

119. Write a short note on the use of Dirac delta function in the theory of Green's function.

120. Describe the relation between Dirac delta function and Green's function

121. Give the eigen value-eigen function equation and explain its significance based on Green's function.

122. Write a short note on Green's function and obtain the relation with Dirac delta function and Green's function.

123. What is meant by Green's function. Establish its relation with Dirac delta function

124. Contrast the interpretation of Green's function in Sturm-Liouville eigenvalue equation with ordinary inhomogenous Sturm-Liouville equation.

B Part

125. State and Prove Cauchy Riemann conditions. Distinguish between analytic and harmonic functions with proper examples.

126. State and prove Cauchy's integral formula. Explain how it can be used to find the derivative of a function

127. Explain the concept of poles. Prove Cauchy's residue theorem and obtain a formula to find the residue.

128. State and prove Cauchy's integral theorem. Illustrate with a suitable example.

129. Derive Cauchy's integral theorem. Compare it with Cauchy's integral formula.

130. Discuss the concept of singularities. Obtain the residue theorem

131. State and prove Residue theorem. Obtain an expression for calculating residue.

132. Describe the symmetry transformations of a square and deduce its multiplication table. Also, find the classes of this group

133. Discuss the symmetry operations of an equilateral triangle. Show that the symmetry operations form a group. Find the classes corresponding to this group.

134. Describe the symmetry transformations of an equilateral triangle and deduce its multiplication table. Also, find the classes of this group

135. Explain the various symmetry transformations of a square. Show that these symmetry transformations form a group and obtain its multiplication table.

136. Show that the z- component of orbital angular momentum is the generator of the rotation function.

137. Obtain the generators of SO(2) and SO(3) groups. Explain the relation between orbital angular momentum and rotation function

138. Discuss the properties of generators of continuous groups and derive the generators of SU(2) and SO(3).

139. Compare Homomorphism and Isomorphism. Explain how SU(2) and SO(3) groups are homomorphic to each other.

140. Explain the homomorphism of groups. Establish the homomorphism between SU(2) and SO(3) groups.

141. Obtain the Euler equation to find the stationary value of a function. Give any two examples

142. Derive the Euler equation with constraints and discuss any two applications.

143. Explain the concept of variation and hence determine the optical path near event horizon of a blackhole.

144. Explain the concept of variation and using it, solve the soap film problem.

145. Derive Euler's equation by applying variational principle. How can it be generalized for the case of several dependent and several independent variables?

146. Discuss the concept of variation for problems involving constraints. Hence, determine the critical angle at which a particle flies off while sliding on a cylindrical surface.

147. Explain Rayleigh-Ritz variational technique. How is it used to compute eigenfunctions and eigenvalues

148. Explain the Euler equation with respect to constraints. Analyse the sliding of a particle on a cylindrical surface using it.

149. What is Lagrangian multiplier in calculus of variation? Illustrate with example. Mention the advantages and specify the case at which it fails.

150. Explain the concept of variation. Using it, determine the radial position of a particle on a frictionless horizontal surface subject to constraints, as a function of time

151. Obtain the Schrodinger equation from variational principle.

152. Derive a Fredholm integral equation corresponding to $y''(x) - y(x) = 0$, $y(1) = 1$, $y(-1) = 1$, (a) by integrating twice,(b) by forming the Green's function.

153. Derive a Fredholm integral equation corresponding to $y''(x) + a_1y'(x) + a_2y(x) = 0$. The boundary conditions are $y(0)=y(1)=0$

154. Convert the Schrodinger equation into Fredholm integral equation of second kind

155. Transform the linear oscillator equation $y''+\omega^2y=0$ into an integral equation. The boundary conditions are $y(0)=0$ and $y(b)=0$. What are the properties of the kernel of this equation?

156. Transform the linear second order ODE $y''+A(x)y'+B(x)y= g(x)$ into an integral equation. Assume the initial conditions to be $y(a)=y_0$ and $y'(a)=y_0'$.

157. Explain the fourier transform method for solving integral equation. Illustrate with a suitable example.

158. Discuss the technique of seperable kernel for solving integral equation.

159. Explain the theory of Neumann series solution for solving an Fredholm integral equation.

160. Explain Green's function and its applications. Find the eigenfunction expansion for Helmholtz equation

161. Deduce the eigen function expansion of Green's function and comment on its symmetry property.

162. Starting from the Sturm - Liouville equation, deduce the solution for Green's function.

163. Develop a solution for Green's function for a nonhomogenous Sturm-Liouville equation.

164. Discuss the properties of Green's function. How it can be used to solve a non-homogenous Sturm-Liouville equation?

165. Discuss the properties of Green's function in detail and establish a relation between Green's function and Dirac Delta function.

C Part

166. Show that the derivative of f(z) with respect z* does not exist unless f(z) is a constant.

167. Using $f(r,\theta) = R(r,\theta)e^{i\Theta(r,\theta)}$. Obtain the Cauchy-Riemann conditions in polar form.

168. Find the analytic function for the following cases. (a) $u(x,y) = x^3 - 3xy^2$ (b) $v(x,y) = e^{-y}\sin x$.

169. Evaluate $\int \frac{e^z}{(z^2+\pi^2)^2} dz$ on a circle, |z|=4.

170. Using Cauchy integral formula obtain the expression for derivatives of a function.

171. Evaluate $\int \frac{e^{iz}}{z^3} dz$ on a circle, |z|=2.

172. Evaluate the closed integral $\int \frac{dz}{(z-a)^n}$ on a simple closed contour.

173. Obtain the Laurent expansion of $\frac{z\,e^z}{(z-1)}$ about z = 1

174. Expand $f(z) = \frac{z+1}{z+3}$ in a Laurent series valid for the region 1<|z|<3.

175. Find the Taylor expansion of $ln\,(1+z)$.

176. Show that e^{z^2} has an essential singularity at infinity. {Hint: use Taylor expansion}

177. Evaluate the definite integral $I = \int_0^{2\pi} \frac{d\theta}{1+\epsilon\cos\theta}$, where $|\epsilon| < 1$

178. Evaluate the integral,$I = \int_0^{\infty} \frac{\sin x}{x} dx$

179. Use the residue theorem to evaluate the integral $I = \int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x} dx$ with a>0.

180. Evaluate the definite integral $I = \int_0^{2\pi} \frac{d\theta}{1+\epsilon\cos\theta}$, where$|\epsilon| < 1$

181. Evaluate the integral $I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

182. Evaluate the integral $I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

183. Evaluate the integral,$I = \int_0^{\infty} \frac{\sin x}{x} dx$

184. Check whether the set of the following four matrices forms a group under matrix multiplication.
$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}, \begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}, \begin{bmatrix}-1 & 0\\1 & -1\end{bmatrix}$

185. What is a group. Check whether the set {1,-1,i,-i} is a group.

186. Obtain the multiplication table of the group {1,-1,i,-i}.

187. Show that the set of all unitary matrices of order n is a group under matrix multiplication.

188. Show that the set of all orthogonal matrices of order n is a group under matrix multiplication.

189. Prove that the set of all positive real numbers including zero is a group under ordinary addition.

190. Obtain the multiplication table of the symmetry group of an equilateral triangle.

191. Starting from an element subject to the only condition $A^n = E$, the identity element, such that n is the smallest integer satisfying the condition, generate the group.

192. Check whether the set of all positive integers including zero is a group under ordinary addition.

193. Prove that the n^{th} root of unity is an abelian group.

194. Show that the set of all non – singular matrices of order n x n is a group under multiplication.

195. Show that 2x2 matrices of the form
$\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ form a group under matrix multiplication.

196. Prove that the cube root of unity is an abelian group.

197. Prove that the set of all positive integers does not form a group under ordinary multiplication.

198. Starting from two elements A and B, subject to the condition $A^2 = B^2 =(AB)^2 = E$, generate a group.

199. Show that the symmetry transformations of a square constitute a group.

200. Show that the set of all orthogonal matrices of order 2 of the form
$\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$
is a group under matrix multiplication.

201. Obtain the symmetry group of an equilateral triangle.

202. Check whether the set of all integers is a group under the binary composition of addition.

203. Find the conjugate elements of m_x of the C_{4v} group.

204. Find out the class structure of the elements C_{2a}, C_{2b}, C_{2c} of the C_{3v} group(symmetry group of an equilateral triangle).

205. Find the conjugate element of $C_2{}^a$ of the symmetry group of an equilateral triangle.

206. Find out the class structure of the element C_3 of the C_{3v} group(symmetry group of equilateral triangle).

207. Prove that if an element A is conjugate with C, another element B is conjugate with C, then A and B must be conjugate to each other.

208. Show that no element can be common between two distinct classes.

209. Find the conjugacy classes of the group C_{4v}.

210. Prove that if an element is conjugate with another element of a group, then the second element is conjugate with the first element.

211. Prove that each element of a cyclic group is a class.

212. Find out the conjugate element of C_3 of the symmetry group of equilateral triangle.

213. Find out the class structure of the element E of the C_{3v} group(symmetry group of an equilateral triangle).

214. Prove that an element of a group is conjugate with itself transformed by identity element.

215. Combine the two cyclic groups of order 2 and 3 to form a new group.

216. Find out the factor group containing invariant subgroup H= {E, C3, $C3^{-1}$} of C_{3v} group.

217. Explain homomorphism. Illustrate with an example.

218. List out subgroups of C_{4v} and identify the normal subgroups.

219. What the conditions for taking the direct product of two groups? Explain the advantage of direct product of groups using an appropriate example.

220. Obtain the subgroups of the symmetry group of an equilateral triangle.

221. Give an example of a group which has a subgroup. Construct a multiplication table for its elements.

222. Show that the group generated by two commuting operators A and B such that $A^2=B^3=E$, where E is the identity element, is cyclic. What is its order?

223. Find out the distinct cosets of the subgroup $\{E, C_2^a\}$.

224. Prove that a group of prime order is always cyclic.

225. Prove that two right cosets of a subgroup of a given group are either equal or have no elements in common

226. Find out subgroups and cosets of the symmetry group of an equilateral triangle.

227. Prove that every cyclic group is an abelian group.

228. Prove that $(1)^{1/3}$ is a cyclic group.

229. Show that every subgroup of a cyclic group is cyclic.

230. List out the subgroups of C_{3v} and identity which are invariant group.

231. If a group H of order h is a subgroup of a group G of order g, show that g is an integral multiple of h.

232. Find out the direct product of subgroups $H_1=\{E,mx\}, H_2=\{E,my\}$

233. Prove that the group of all positive real numbers under multiplication is isomorphic to the group of all real numbers under addition.

234. Prove that 2 right cosets of a subgroup in a given group are either equal or else have no elements in common.

235. Obtain the distinct groups of order 1,2 and 3.

236. Prove that 2 left cosets of a subgroup in a given group are either equal or else have no elements in common.

237. Prove that (i) group of order two is always cyclic (ii) group of order three is always cyclic (iii) group of order 4 may or may not be cyclic.

238. Apply Euler equation to find the shortest distance between two points in Euclidean space.

239. Determine the optical path near the event horizon of a black hole

240. Find the equation to a line connecting two parallel coaxial wire circles such that the revolving about the x- axis produces the minimum surface area.

241. Find the minimum surface area of a soap film.

242. Determine the shortest distance between two points in the Euclidean xy plane

243. Obtain the Lagrangian equation of motion using variational principle.

244. Obtain the Lagrangian equation for a particle moving in the cartesian 1 D space.

245. Obtain the Laplace equation using variational concept.

246. A rectangular parallelepiped is inscribed in an ellipsoid of semiaxes a, b, and c. Maximize the volume of the inscribed rectangular parallelepiped. Find the ratio of the maximum volume to the volume of the ellipsoid.

247. Obtain an equation for a particle sliding on a cylindrical surface.

248. Find the ratio of R (radius) to H (height) that will minimize the total surface area of a right-circular cylinder of fixed volume.

249. Obtain an equation for the simple pendulum of length l swinging in an arc

250. From the lens equation 1/u + 1/v = 1/f, find the minimum object image distance (u+v) for the formation of real image, applying Lagrangian multipliers.

251. Find the eigen function and eigen value corresponding to a vibrating string clamped at x=0 and 1.

252. Determine the critical angle at which a particle flies off while sliding on a cylindrical surface.

253. Find the acceleration of a cylinder of radius a and mass m rolling on an inclined plane of length l at an angle of ψ with the horizontal surface.

254. A rectangular parallelepiped is inscribed in an ellipsoid of semiaxes a, b, and c. Maximize the volume of the inscribed rectangular parallelepiped. Show that the ratio of the maximum volume to the volume of the ellipsoid is $2/\pi\sqrt{3} \approx 0.367$.

255. Find the shape of a box that will minimize energy of a quantum mechanical particle subject to the condition that the volume of the box is constant.

256. Determine the ratio h/r of a right circular cylinder of radius r and height h that will minimize its surface area for a fixed enclosed volume.

257. Derive the volterra integral equation corresponding to $y''(x) - y(x) = 0$, $y(0) = 0$, $y(0) = 1$.

258. Transform the linear oscillator equation $y'' + \omega^2 y = 0$ into an integral equation with $y(0)=0$ and $y'(0)=1$.

259. Solve $\varphi(x) = x + \int_0^x (t-x)\varphi(t)dt$ using Laplace transform solution method.

260. Solve the integral equation $f(x) = \int \frac{\varphi(t)}{(1-2xt+x^2)^{1/2}} dt$ for ϕ(t). Given $f(x) = x^{2s}$.

261. Solve $\varphi(x) = \lambda \int_{-1}^{1} (t+x)\varphi(t)dt$ using seperable Kernel method.

262. Solve $\varphi(x) = x + \frac{1}{2} \int_{-1}^{1} (t-x)\varphi(t)dt$ using Nuemann series solution.

263. Show that Green's function is symmetric using Eigenfunction expansion method.

264. Explain the eigenfunction expansion for a Green's function.

265. Find the Green's function for $Ly(x) = \frac{d^2y(x)}{dx^2} + y(x)$ with boundary conditions y(0)=0 and y'(1)=0.

266. Find the Green's function for operator $L = \frac{d^2}{dx^2}$ with boundary conditions y(0)=0 and y'(1)=0.

267. Solve the eigenvalue equation for the harmonic oscillator equation and find the corresponding eigenfunction. Use them to obtain the expression for Green's function using eigenfunction expansion.

268. FInd the Green's function for $y''(x) + \lambda y(x) = 0$ with boundary conditions y(0)=y(1)=0.

D Part

E Part