23P254 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - PG)

(Regular/Supplementary)

CC19P MST2 C07 / CC22P MST2 C07 - ESTIMATION THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define suffiency. State and prove factorization theorem for sufficiency.
2. Explain one parameter exponential family of distributions. Prove that geometric distribution distribution is a member of one parameter exponential family.
3. Define Fisher information. Find the Fisher Infromation of Caushy distribution.
4. Explain the method of percentiles for estimation of parameters.
5. If $f(x) = 1, \theta - \frac{1}{2} \leq x \leq \theta + \theta + \frac{1}{2}$, obtain the M.L.E of θ.
6. Explain Loss function and different types of loss function.
7. Let $X \sim U(0, \theta)$. Obtain an unbiased C.I for θ.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Explain Point estimation and Interval estimation. Also explain the desirable properties of a good estimator.
9. Define MVUE. Prove that MVUE is unique.
10. Define Cramer Rao Lower Bound. Find Cramer-Rao lower bound for variance of the unbiased estimator of θ with $f(x;\theta) = \theta(1-\theta), x = 0, 1, 2, \ldots,$ and $0 < \theta < 1$.
11. Define Consistency. Let $x \sim P(\lambda)$. Check whether $T = \frac{2}{n(n+1)} \sum_{i=1}^{n} X_i$ is a consistent estimator of λ.
12. Let $x_1, x_2, \ldots, x_n$ be a random sample of size 'n' from $N(\mu, \sigma^2)$ .if $T_n = \bar{x}$, show that T_n is a CAN estimator.
13. Show that under squared loss function,the bayes estimator is the mean of posterior distribution.

14. Obtain the shortest considence inetrval for variance of a normal distribution based on 'n' observation, with confidence coefficient $(1-\alpha)$.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. i) State and prove Rao-Blackwell theorem

    ii) Let $x_1, x_2, \ldots, x_n$be a random sample from $N(\theta, 1)$.Find the UMVUE of θ and θ^2.

16. i) State and prove sufficient condition for consistency of a estimator.

    ii) Find a consistent estimator $e^{-\lambda}$ of if $x \sim P(\lambda)$.

17. Explain Cramer family. State and prove Cramer-Huzurbazar theorem.

18. i) Define pivote.Describe the method of construction of confidence interval using pivot.

    ii) Find the $100(1-\alpha)\%$ shortest length confidence inetrval for variance of normal disrtibution based on 'n' observation.

(2 × 5 = 10 Weightage)

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