23P256 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST2 C09 / CC22P MST2 C09 - TESTING OF STATISTICAL HYPOTHESES

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. a) Define most powerful test
   b) Let $\phi(x)$ be a most powerful test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$
   Show that $\phi^*(x) = 1 - \phi(x)$ is a most powerful test for $H_0 : \theta = \theta_1$ against $H_1 : \theta = \theta_0$
2. Let p be the probability that a given die shows an even number. To test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{1}{3}$ the following procedure is adopted. Toss the die twice and accept H_0 if both dies shows an even number. Define power function. Find probability of type I error and power of test.
3. Explain:
   i) uniformly most powerful unbiased test
   ii) If a size α UMP test exists, then show that it is uniformly most powerful unbiased
4. Define a) α similar test b) Uniformly most powerful α similar test
5. Write a short note on sign test for median of a population has a specified value.
6. Explain Kolmogorov-Smirnov two sample test.
7. Let N be the number of items taken to a sample in an SPRT. Show that moment generating function of N exists

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. When will you say that a family of distributions possess MLR property ? Verify the family $C(1, \theta)$ possess MLR property.
9. Let $X_1, X_2, \ldots, X_n$ be a sample taken from Bernoulli population $B(1, \theta)$. Using Karlin Rubin theorem derive the UMP test for testing $H_0 : \theta \leq \theta_0$ versus $H_0 : \theta > \theta_0$ based upon the sample taken.

10. Define Complete statistic. Let $X_1, X_2, \ldots X_n$ be a sample from a $B(1,p)$ population Define $Y = \sum_{i=1}^{n} X_i$ Show that Y is a complete Statistic.

11. Explain chi-square test for homogeneity.

12. Define OC function of SPRT and derive its expression.

13. Determine the boundary points A and B of SPRT in terms of the strength of the test.

14. Let $X \sim P(\lambda)$, consider $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (\lambda > 0)$. Derive SPRT and find OC function.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Consider a sample $X_1, X_2, \ldots, X_n$ taken from a population following $N(0, \sigma^2)$ for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 = \sigma_1^2 (\sigma_1^2 \neq \sigma_0^2)$. Obtain the most powerful test using NP approach.

16. State and Prove Neyman Pearson Lemma for the most powerful test.

17. Write a note on:
    (a) Mann-Whitney Wilcoxon test.
    (b) Sign test for two sample
    (c) Median test.

18. Develop SPRT for testing for testing $H_0 : \mu = \mu_0$ agaist $H_1 : \mu = \mu_1 (\mu_1 > \mu_0)$ for observations taking from Normal distribution $N(\mu, \sigma^2)$, where σ^2 is known.

(2 × 5 = 10 Weightage)

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