**23P201** (Pages: 2) Name: ...............................

Reg.No: ...............................

**SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024**

(CBCSS - PG)

(Regular/Supplementary/Improvement)

**CC19P MTH2 C06 - ALGEBRA - II**

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

**Part A**

Answer any ***all*** questions. Each question carries 1 weightage.

1. Show that $\mathbb{Q}[x]/\langle x^2-2\rangle$ is a field.
2. Prove that $\mathbb{R}(i) \cong \mathbb{C}$
3. Show that a regular 9-gon is not constructible.
4. Prove that finite extension of a finite field is finite.
5. Prove that any two algebraic closures of a field $F$ are isomorphic.
6. Find the splitting field of $\{x^2-2, x^2-3\}$ over $\mathbb{Q}$
7. State Primitive Element Theorem.
8. Prove that $\mathbb{Q}(\sqrt{2})$ is an extension of $\mathbb{Q}$ by radicals.

**(8 × 1 = 8 Weightage)**

**Part B**

Answer any ***two*** questions each unit. Each question carries 2 weightage.

**UNIT - I**

9. If $E$ is finite extension field of a field $F$, and $K$ is a finite extension field of $E$, then show that $K$ is a finite extension of $F$ .and $[K:F]=[K:E][E:F]$.
10. Find a basis and dimension of for $\mathbb{Q}(\sqrt{2},\sqrt{3})$ over $\mathbb{Q}$.
11. Let $E$ be an extension field of $F$, then show that $\overline{F}_E=\{\alpha \in E : \alpha \text{ is algebraic over } F\}$ is a subfield of $E$.

**UNIT - II**

12. A finite field $GF(p^n)$ of $p^n$ elements exist for every prime power $p^n$.
13. If $E$ is a finite extension of $F$, then $\{E:F\}$ divides $[E:F]$.

14. If $E$ is a finite extension of $F$, then prove that $E$ is separable over $F$ if and only if each $\alpha$ in $E$ is separable over $F$.

**UNIT - III**

15. Let $K$ be a finite normal extension of $F$, and let $E$ be an extension of $F$, where $F \leq E \leq K \leq \overline{F}$. Then prove that

    (a) $K$ is a finite normal extension of $E$

    (b) $G(K/E)$ is precisely the subgroup of $G(K/F)$ consisting of all those automorphisms that leave $E$ fixed.

16. State Main Theorem of Galois Theory.

17. Find $\Phi_6(x)$ over $\mathbb{Q}$.

**(6 × 2 = 12 Weightage)**

**Part C**

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let $R$ be a commutative ring with unity. $M$ is an ideal in $R$. State and prove necessary and sufficient conditions for $M$ becomes a maximal ideal.

19. State and Prove Kroneckers Theorem.

20. Let $F$ be a finite field of characteristic $p$. Then show that the map $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$ for $a \in F$, is an automorphism, the Frobenius automorphism in $F$. Also prove that $F_{\{\sigma_p\}} \simeq \mathbb{Z}_p$.

21. Let $F$ be a field of characteristic 0 and let $a \in F$. If $K$ is the splitting field of $x^n - a$ over $F$, then show that $G(K/F)$is a solvable group.

**(2 × 5 = 10 Weightage)**

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