23P202 (Pages : 2) Name : ....

Reg. No : ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024 (CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C07 - REAL ANALYSIS - II

(Mathematics)

(2019 Admission onwards)

Time: Three hours Maximum : 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Check whether the collection of all open sets and closed sets in $\mathbb{R}$ is a σ-algebra or not.
2. Prove that a countable set has outer measure zero.
3. Show that a continuous real-valued function defined on a measurable set is measurable.
4. Show that strict inequality occurs in Fatou's lemma.
5. Let f be integrable over E. Suppose A and B are disjoint measurable subsets of E. Prove that $\int_{A\cup B} f = \int_A f + \int_B f$.
6. Let f be integrable over E. Prove that for each $\epsilon > 0$, there is a set of finite measure E_0 for which $\int_{E\sim E_0} |f| < \epsilon$.
7. Show that a Lipschitz function on $[a, b]$ is of bounded variation on $[a, b]$.
8. Define a rapidly Cauchy sequence and give an example.

(8 × 1 = 8 Weightage)

PART B

Two questions should be answered from each unit. Each question carries 2 weightage.

UNIT I

9. Prove that outer measure is countably subadditive.
10. Show that the Cantor set is a closed, uncountable set of measure zero.
11. State and prove Lusin's theorem.

UNIT II

12. State and prove the bounded convergence theorem.

13. Let f be integrable over E and $\{E_n\}_{n=1}^{\infty}$ a disjoint countable collection of measurable subsets of E whose union is E. Prove that $\displaystyle\int_E f = \sum_{n=1}^{\infty} \int_{E_n} f$.

14. (a) Define convergence in measure.

    (b) Show that if $\{f_n\} \to f$ in measure on E, then there is a subsequence $\{f_{n_k}\}$ that converges pointwise a.e. on E to f.

UNIT III

15. Prove that a monotone function on an open interval (a, b) is continuous except possibly at a countable number of points in (a, b).

16. Show that a function f on a closed, bounded interval $[a, b]$ is absolutely continuous on $[a, b]$ if and only if it is an indefinite integral over $[a, b]$.

17. State and prove Young's inequality.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Show that every interval is measurable.

    (b) State and prove the simple approximation theorem.

19. (a) State and prove the monotone convergence theorem.

    (b) Prove that a bounded function defined on a set of finite measure E is Lebesgue integrable over E if and only if it is measurable.

20. State and prove the Vitali covering lemma.

21. (a) Show that a convex function φ on (a, b) is differentiable except at a countable number of points and its derivative φ' is an increasing function.

    (b) State and prove Riesz-Fischer theorem.

(2 × 5 = 10 Weightage)

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