23P204 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C09 - ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Verify that $\cos x = \left[\lim_{a\to\infty} F(a, a, \frac{1}{2}, \frac{-x^2}{4a^2})\right]$.

2. Determine the nature of the point $x = \infty$ for the Legendre's equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0.$

3. Find the first two terms of the Legender series of $f(x) = e^x$.

4. Show that $\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases}$ and $\begin{cases} x = e^{-t} \\ y = -e^{-t} \end{cases}$ are independent solutions of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = x + 2y \\ \dfrac{dy}{dt} = 3x + 2y \end{cases}$ on every closed interval.

5. Find the critical points and then describe the phase portrait of the system $\dfrac{dx}{dt} = -x, \quad \dfrac{dy}{dt} = -y.$

6. Determine the nature and stability properties of the critical point $(0,0)$ of the linear autonomous system $\dfrac{dx}{dt} = -3x + 4y, \quad \dfrac{dy}{dt} = -2x + 3y.$

7. Using Picard's method of successive approximation, solve the initial value problem $y' = y, y(0) = 1$ (start with $y_0(x) = 1$).

8. Prove that the plane curve of fixed perimeter and maximum area is the circle.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Show that $\tan x = x + \frac{1}{3} + \frac{2}{15}x^5 + \cdots$ by solving the differential equation $y' = 1 + y^2$.

10. Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x.

11. Show that $(x-t)\sum_{n=0}^{\infty} P_n(x)t^n = (1-2xt+t^2)\sum_{n=1}^{\infty} nP_n(x)t^{n-1}$.

UNIT - II

12. Describe about different types of critical points.

13. Show that $(0,0)$ is an asymptotically stable critical point for the system
$\frac{dx}{dt} = -3x^3 - y,\ \ \frac{dy}{dt} = x^5 - 2y^3$.

14. Verify that (0, 0) is a simple critical point for the system
$\frac{dx}{dt} = x + y - 2xy,\ \ \frac{dy}{dt} = -2x + y + 3y^2$ and determine its nature.

UNIT - III

15. Let $u(x)$ be any nontrivial solution of $u'' + q(x)u = 0$, where $q(x) > 0$ for all $x > 0$. If $\int_1^{\infty} q(x)dx = \infty$, then prove that $u(x)$ has infinitely many zeros on the positive x axis.

16. State and prove Sturm comparison theorem.

17. Show that $f(x,y) = xy^2$ satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$, but does not satisfy a Lipschitz condition on any strip $a \le x \le b$ and $-\infty < y < \infty$.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Find two independent Frobenius series solutions of the differential equation
$x^2y'' - x^2y' + (x^2 - 2)y = 0$.

19. (i) Derive Bessel's function of first kind.
(ii) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

20. State and prove the orthogonality property of Bessel functions.

21. Derive Euler's equation for an extremal and find the extremal of the integral $\int_{x_1}^{x_2} \frac{\sqrt{1+(y')^2}}{y} dx$.

(2 × 5 = 10 Weightage)

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