23P205 (Pages: 2) Name: .....
Reg. No: .....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024
(CBCSS - PG)
(Regular/Supplementary/Improvement)
CC19P MTH2 C10 - OPERATIONS RESEARCH
(Mathematics)
(2019 Admission onwards)

Time: 3 Hours Maximum : 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define convex function.
2. Explain Degeneracy in linear programming problem.
3. Prove that dual of the dual is primal.
4. Explain simplex multipliers.
5. Explain Degeneracy in transportation problem.
6. Define the terms chain, circuit, cycle and path.
7. Explain integer programming.
8. Find the saddle point, if it exists, for the payoff matrix $\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semidefinite, then prove that $f(X)$ is a convex function.
10. Solve by simplex method Maximize $z = 7x_1 + 5x_2$
    subject to $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 12$, and $x_1 \geq 0, x_2 \geq 0$
11. Solve the LPP using Big M Method Minimize $2x_1 - 3x_2 + 6x_3$
    subject to $3x_1 - 4x_2 - 6x_3 \leq 2$, $2x_1 + x_2 + 2x_3 \geq 11$,
    $x_1 + 3x_2 - 2x_3 \leq 5$, and $x_1, x_2, x_3 \geq 0$

UNIT II

12. Write the dual of the following LPP. Minimize $x_1 - 3x_2 - 2x_3$

    subject to $2x_1 - 4x_2 \geq 12,\ 3x_1 - x_2 + 2x_3 \leq 7,$
    $-4x_1 + 3x_2 + 8x_3 = 10,\ x_1, x_2 \geq 0,\ x_3$ unrestricted

13. Solve by dual simplex method Minimize $z = x_1 + 3x_2 + 2x_3$
    subject to $4x_1 - 5x_2 + 7x_3 \leq 8,$
    $-2x_1 + 4x_2 - 2x_3 \leq -2, \quad x_1 - 3x_2 + 2x_3 = 2, \quad x_1, x_2, x_3 \geq 0.$

14. Find the optimum solution of the following transportation problem.

	D_1	D_2	D_3	D_4	
O_1	3	2	5	4	25
O_2	4	1	7	6	35
O_3	7	8	3	5	30
	10	18	20	42	

UNIT III

15. Explain Cutting palne method.
16. Explain the algorithm of maximum flow problem.
17. Solve graphically the game whose payoff matrix is $\begin{bmatrix} 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix}$

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Solve the following problem using two phase method. Maximize $Z = 4x_1 + 5x_2$

    subject to $2x_1 + x_2 \leq 6, \quad x_1 + 2x_2 \leq 5, \quad x_1 + x_2 \geq 1, \quad x_1 + 4x_2 \geq 2$ and $\quad x_1 \geq 0,\ x_2 \geq 0$

19. Find the minimum path from v_1 to v_8 in the graph with arcs and arc lengths given below:

$(1,2)$	$(1,3)$	$(1,4)$	$(2,3)$	$(2,6)$	$(2,5)$	$(3,5)$	$(3,4)$	$(4,7)$
1	4	11	2	8	7	3	7	3

$(5,6)$	$(5,8)$	$(6,3)$	$(6,4)$	$(6,7)$	$(6,8)$	$(7,3)$	$(7,8)$
1	12	4	2	6	10	2	2

20. Solve by simplex method: Maximize $Z = -5x_1 + 13x_2 + 5x_3$
    subject to $12x_1 + 10x_2 + 4x_3 \leq 90,\ -x_1 + 3x_2 + x_3 \leq 20,\ x_1 \geq 0,\ x_2 \geq 0.$
    Use the sensitivity analysis find an optimal solution when right side of the second constraint is changed to 30.
21. State and prove fundamental theorem of rectangular games.

(2 × 5 = 10 Weightage)

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